NUMERICAL ANALYSIS OF MULTIGROUP NEUTRON FLUX IN A BARE FAST REACTOR

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THESIS

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ABSTRACT

This study consists of the development of a computer program to numerically solve the space and energy dependent multigroup neutron diffusion equations in a bare homogeneous fast reactor core or reactor material assembly.

The resulting program is unique in that it was designed for future use by Naval Postgraduate School students undertaking experimental studies in neutron diffusion with limited time to determine numerical solutions for verification of their results.

The equations are solved iteratively in cylindrical geometry using a point successive overrelaxation technique. Convergence between successive iterations was less than 10^{-6} after fifty iterations.

The program was tested using ANL three group data. Flux shapes and energy spectra were determined for a typical fast reactor core and for a solid iron cylinder with a source at its center. The program was also used to determine criticality.

Computation times were from one to ten minutes with less than 150K words of core storage using the IBM 360/67.

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TABLE OF SYMBOLS

$\bar{r}$	position vector
v	speed
$\bar{\Omega}$	direction of motion of neutron
t	time
η	neutron density
Σ_s	macroscopic scattering cross section
Σ_a	macroscopic absorption cross section
Σ_t	macroscopic total cross section
q	slowing down density
u	lethargy variable
E	energy variable
ϕ_g	neutron flux of the gth energy group
D_g	diffusion coefficient of the gth energy group
$\bar{\Sigma}^g_{tr,g}$	macroscopic transport cross section of the gth energy group
N^z	atomic abundance of the Zth element or isotope
$\Sigma_{f,g}$	macroscopie fission cross section of the gth energy group
$\Sigma_{c,g}$	macroscopic capture cross section of the gth energy group
$\Sigma_{in,g}$	macroscopic inelastic removal cross section of the gth energy group
$\Sigma_{er,g}$	macroscopic elastic removal cross section of the gth energy group
χ_g	the fraction of fission neutron born into group g
ν	average number of neutrons emitted per fission
Sg	neutron source
B^2	buckling
r	radial position index
z	axial position index

Θ	angular position index
h	mesh spacing
$\overline{\overline{W}}$	source matrix
$\mathfrak{Z}$	scale factor
K_{eff}	effective multiplication constant
ϵ_1	inner iteration convergence criterion
ϵ_2	outer iteration convergence criterion
w	overrelaxation factor
x, y, z	Cartesian coordinates
γ	angular mesh spacing
i, j	Cartesian position indices

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I. INTRODUCTION

A. BACKGROUND

It has been found that the energy-dependent diffusion equation provides a fairly accurate description of the neutron behavior in fast reactor systems [1].

One way of conveniently using this equation is the now familiar multigroup method which was developed for reactor calculations by F. L. Friedman, A. M. Weinberg, and J. A. Wheeler [2] about 1947. Since that time, many solutions to the age-diffusion equation have resulted from the use of the multigroup method. Some analytical and semianalytical solutions have been carried out for a few group equations [3] but most of the work done has been numerical due mainly to the need for large numbers of energy groups, spatial points, and spatial regions.

The numerical solutions to unique problems and classes of problems have evolved in the form of computer programs or codes. The degree of complexity and sophistication of these programs has roughly paralleled that of the digital computer. At this point in time, the literature abounds with numerous and varied codes and references thereto. Sangren [4] states that by 1959 there existed in the United States more than 300 nuclear reactor codes for digital computers. Various groups have classified and categorized these codes [4]. In 1961 the American Nuclear Society and the Argonne National Laboratory established a code center for the purpose of collecting and disseminating the multitude of codes that existed and for the future codes which were sure to be forthcoming [5]. Around 1965 the European counterpart to this center was established [5].

Most of the more recent codes (those published after 1965) examined in connection with this study are somewhat specialized

along the following lines: a) number of energy (lethargy) groups; b) number of spatial dimensions; c) number of regions; d) cross-section data input; e) geometry; f) number of mesh points; g) method of equation solution; h) finite difference approximation and i) many other features related to poisons, burn-up, perturbations, computer system, and others.

B. OBJECTIVES

This study was undertaken to provide a relatively simple-to-apply multigroup diffusion program for application to fast reactor cores and materials. In the future students at the Naval Postgraduate School will be able to use this program in conjunction with experimental neutron diffusion studies. In view of the large number of complex codes that exist and the difficulties related to modifying these codes for use on a specific problem and a specific computer system, it is anticipated that valuable research time may be saved by use of this program.

Once the program was developed, initial testing would be done using a subcritical assembly. The first phase of this test would involve the solution of the three group diffusion equation in a typical fast core composition. From this, the energy and spatial dependence of the flux could be determined and comparisons made with the solution to the diffusion problem wherein space and energy dependence are treated as separable variables.

The second phase of the initial test would involve the problem of the typical core assembly with a Pu-Be neutron source located at its center. This would then be modified to a solid iron cylinder with a centrally located Pu-Be neutron source. The flux shapes and energy spectra resulting from this study could then be used for comparison with experimental data resulting from local research. Experiments in this area are currently underway with applications

to fast reactor cross section data evaluation, materials and shielding reference [6].

Using the program to determine a critical composition and size for the typical fast reactor was another desired goal. In this area, a study was to be made on the effect mesh spacing has on criticality if any.

Other objectives were to compare the energy spectra of the three and the sixteen group solutions and to ascertain any difference in the flux shapes and spectra by using the ANL sixteen group data and the LASL sixteen group data.

II. THEORY

A. THE BOLTZMANN TRANSPORT EQUATION

The conservation or balance of neutrons in a nuclear reactor can be completely described only by simultaneously specifying the distribution in space, time, energy and direction of motion of the neutrons. The general equation governing this balance is called the Boltzmann transport equation and may be written as

$$\text{PRODUCTION} - \text{LEAKAGE} - \text{ABSORPTION} = \frac{\partial n}{\partial t} \quad , \qquad \text{II-1}$$

where $\frac{\partial n}{\partial t}$ is the time rate of change of neutron density.

The neutron density is in general described as

$$\left[\begin{array}{l}\text{number of neutrons at time t in volume} \\ \text{dr about r, whose speeds lie in dv about} \\ \text{v and whose directions of motion (velocity} \\ \text{vectors) lie in the differential solid angle} \\ \text{angle d}\bar{\Omega} \text{ about direction } \bar{\Omega}\end{array}\right] \qquad \text{II-2}$$

and neutron flux is defined as

$$\phi(\bar{r}, v, \bar{\Omega}, t) \equiv v\, n(\bar{r}, v, \bar{\Omega}, t) \qquad \text{II-3}$$

Meghreblian and Holmes [1] describe Equation II-1 verbally as

$$\text{I} \left[\begin{array}{l}\text{number of neutrons at speed in dv about} \\ \text{v moving in direction } \bar{\Omega} \text{ which appear} \\ \text{per unit time from source in dr}\end{array}\right]$$

$$+\text{II} \left[\begin{array}{l}\text{number of neutrons of direction } \bar{\Omega} \text{ gained} \\ \text{per unit time in dr from scattering} \\ \text{collisions which scatter the neutron from} \\ \text{all directions } \bar{\Omega}' \text{ to } \bar{\Omega} \text{ and all speeds} \\ \text{to speed in dv about v}\end{array}\right]$$

$$-\text{III}\quad\left[\begin{array}{l}\text{net number of neutrons moving in direction } \bar{\Omega} \text{ at speed in dv about v that are lost per unit time by leakage through the boundaries of dr}\end{array}\right]$$

$$-\text{IV}\quad\left[\begin{array}{l}\text{number of neutrons at speed in dv about moving in direction } \bar{\Omega} \text{ which are removed per unit time from } d\bar{r} \text{ by absorption collisions}\end{array}\right]$$

$$-\text{V}\quad\left[\begin{array}{l}\text{number of neutrons at speed in dv about moving in direction } \bar{\Omega} \text{ which are removed per unit time from } d\bar{r} \text{ by being scattered into a new direction}\end{array}\right]$$

$$=\text{VI}\quad\left[\begin{array}{l}\text{change per unit time in net number of neutrons of direction } \bar{\Omega} \text{ in } d\bar{r}\end{array}\right] \qquad \text{II-4}$$

The first term in Equation II-4 accounts for the rate at which neutrons with speed in dv about v and with direction in $d\bar{\Omega}$ appear from sources in $d\bar{r}$. This term is then written as

$$S(\bar{r}, v, \bar{\Omega}, t)\, d\bar{r}\, dv\, d\bar{\Omega} . \qquad \text{II-5}$$

The second term of Equation II-4 gives the total number of neutrons which appear in the differential element $d\bar{r}\, dv\, d\bar{\Omega}$ per unit time with speed in dv about v and with direction $d\bar{\Omega}$ about $\bar{\Omega}$ by being scattered from all possible speeds v and direction of motion $\bar{\Omega}'$. The number of neutrons in $d\bar{r}\, dv\, d\bar{\Omega}$ which experience this scattering is

$$\Sigma_s(v')\,\phi(\bar{r}, v', \bar{\Omega}', t)\, d\bar{r}\, dv'\, d\bar{\Omega}' . \qquad \text{II-6}$$

The probability that the scattering collision results in the neutron having speed in dv about v and direction $d\bar{\Omega}$ about $\bar{\Omega}$ is defined by

$$f(v, \bar{\Omega}; v', \bar{\Omega}') \equiv$$

[probability that a neutron with an initial speed v and direction of motion $\bar{\Omega}'$, when scattered, emerges from the collision with a speed in dv about v and direction of motion in $d\bar{\Omega}$ about $\bar{\Omega}$.] II-7

If Equations II-6 and II-7 are multiplied together and integrated over all possible v' and $\bar{\Omega}'$, the result is the total gain of neutrons in the differential element $d\bar{r}\,dv\,d\bar{\Omega}$ due to the scattering in of neutrons from higher speeds and different directions of motion written

$$\int_{v'\bar{\Omega}'} \phi(\bar{r}, v, \bar{\Omega}, t)\, \Sigma_s(v')\, f(v, \bar{\Omega}; v', \bar{\Omega}') \, d\bar{r}\, dv' d\bar{\Omega}'\, d\bar{\Omega}\, dv \,. \qquad \text{II-8}$$

The third term of Equation II-4 represents the net loss of neutrons from $d\bar{r}\,d\bar{\Omega}$ due to leakage through the elemental boundaries of $d\bar{r}$ and is equal to

$$\bar{\Omega} \cdot \nabla \phi \, d\bar{r} d\bar{\Omega} \qquad \text{II-9}$$

A rigorous derivation of Equation II-9 is given in reference [1]. Term number four of Equation II-4 is the absorption term and accounts for all neutrons lost from $d\bar{r}\,dv\,d\bar{\Omega}$ per unit time by absorption; it is given by

$$\Sigma_a(v)\, \phi(\bar{r}, v, \bar{\Omega}, t)\, d\bar{r}\, dv d\bar{\Omega} \,. \qquad \text{II-10}$$

The rate at which neutrons are scattered out of $d\bar{r}\,dv\,d\bar{\Omega}$ into new directions $\bar{\Omega}'$ is the fifth term of Equation II-4 and is written as

$$\Sigma_s(v)\,\phi(\bar{r},v,\bar{\Omega},t)\,d\bar{r}\,dv\,d\bar{\Omega}\,. \qquad \text{II-11}$$

The final term of Equation II-4 is the time-rate of change of the neutron density in $d\bar{r}\,dv\,d\bar{\Omega}$. This term can be written as

$$\frac{\partial n}{\partial t}\,n(\bar{r},v,\bar{\Omega},t)\,d\bar{r}\,dv\,d\bar{\Omega} = \frac{1}{v}\,\frac{\partial}{\partial t}\,\phi(\bar{r},v,\bar{\Omega},t)\,d\bar{r}\,dv\,d\bar{\Omega}\,. \qquad \text{II-12}$$

By placing Equations II-5 through II-12 into the verbal Equation II-4 and dividing out the differential $d\bar{r}\,dv\,d\bar{\Omega}$ the result is the Boltzmann transport equation for the distribution of neutrons of all energies in space and time

$$\frac{1}{v}\,\frac{\partial}{\partial t}\,\phi(\bar{r},v,\bar{\Omega},t) + \Sigma_t(v)\,\phi(\bar{r},v,\bar{\Omega},t) + \bar{\Omega}\cdot\nabla\,\phi(\bar{r},v,\bar{\Omega},t) = S(\bar{r},v,\bar{\Omega},t) + \int_{v'}\int_{\bar{\Omega}'}\phi(\bar{r},v,\bar{\Omega},t)\,\Sigma_s(v')\,f(v,\bar{\Omega};v',\bar{\Omega}')\,dv'\,d\bar{\Omega}'. \qquad \text{II-13}$$

The second term in Equation II-13 is the total loss of neutrons due to scattering out to new spaces and directions plus absorption. The total cross section $\Sigma_t(v)$ is used.

Equation II-13 is perfectly general with the following two exceptions: a) the medium is assumed to be isotropic and homogeneous and b) neutron slowing down is due only to elastic-scattering collisions with nuclei.

of the gth group is defined by the integral

$$\phi_g(\bar{r}) = \int_{E_g}^{E_{g-1}} \frac{\phi(\bar{r},E)}{E}\, dE \qquad \text{II-17}$$

where the upper limit on the integral E_{g-1} is the lower boundary of the energy interval and the lower limit on the integral Eg is the upper boundary of the energy integral. The energy-dependent flux at the point F is $\phi(\bar{r},E)$.

Within each group the diffusion of neutrons is described by the average diffusion coefficient. For simple diffusion theory this is written as

$$D_g = \frac{1}{3\,\Sigma_{tr,g}} \qquad \text{II-18}$$

where $\Sigma_{tr,g}$ is the group macroscopic transport cross section and may be obtained from

$$\Sigma_{tr,g} = \sum_z N^z\, \sigma^z_{tr,g} \quad , \qquad \text{II-19}$$

where the summation is over all elemental components and the group microscopic cross section $\sigma^z_{tr,g}$ in the absence of pronounced resonance effects due to capture and/or scattering with the energy group is given by

$$\sigma^z_{tr} = \frac{\int_{E_{gL}}^{E_{gH}} \sigma^z_{tr,g}(E)\,\phi(E)\,dE}{\int_{E_{gL}}^{E_{gH}} \phi(E)\,dE} \qquad \text{II-20}$$

B. THE AGE-DIFFUSION APPROXIMATION

The age-diffusion approximation to Equation II-13 is developed in reference [1] by expanding the Boltzmann transport equation in spherical harmonics.

In the steady state the resulting first order approximation to Equation II-13 is

$$-D(u)\nabla^2\phi(\bar{r},u) + \Sigma_a(u)\,\phi(\bar{r},u) = S(\bar{r},u) - \frac{\partial}{\partial u}\,q(\bar{r},u). \qquad \text{II-14}$$

Since the change in the slowing down density of neutrons within the lethargy interval du must be balanced by the flow of neutrons scattered into du from elsewhere, the last term of Equation II-14 may be written as

$$-\frac{\partial}{\partial u}\,q(\bar{r},u) = \int \Sigma_s(u' \rightarrow u)\,\phi(u')\,du' \qquad \text{II-15}$$

And Equation II-14 may be rewritten substituting energy for lethargy variable as

$$-D(E)\nabla^2\phi(\bar{r},E) + \Sigma_a(E)\,\phi(\bar{r},E) = S(\bar{r},E) + \int \Sigma_s(E' \rightarrow E)\,\phi(E')\,dE' \qquad \text{II-16}$$

Complete details of this analysis may be found in reference [1].

C. MULTIGROUP FORMULATION

In forming the multigroup diffusion equation, the initial step is to divide the complete neutron energy (lethargy) range into N groups which may or may not be of the same width. The neutron flux

Neutrons may be removed from a particular group g through an absorption interaction described by the group absorption cross section $\Sigma_{a,g}$

$$\Sigma_{a,g} = \Sigma_{f,g} + \Sigma_{c,g} + \Sigma_{in,g} + \Sigma_{er,g} \quad , \qquad \text{II-21}$$

where

$$\begin{aligned}
\Sigma_{f,g} &= \text{fission cross section of group g} \\
\Sigma_{c,g} &= \text{capture cross section of group g} \\
\Sigma_{in,g} &= \sum_{k \neq g} \Sigma_{in,g \to k} = \text{inelastic removal cross section from group g} \\
\Sigma_{er,g} &= \sum_{k \neq g} \Sigma_{er,g \to k} = \text{elastic removal cross section from group g}
\end{aligned} \qquad \text{II-22}$$

The group macroscopic cross sections defined in Equation II-21 above are obtained from the corresponding group microscopic cross sections for each distinct element or isotope in the reactor such that

$$\Sigma_{q,g} = \sum_{Z} N^{Z} \sigma^{Z}_{q,g} \quad , \qquad \text{II-23}$$

where q is the particular reaction and N^Z equals the number of atoms of material z per cubic centimeter.

The multigroup microscopic cross sections for each material are defined in reference [7] as follows:

GROUP CAPTURE

$$\sigma^{Z}_{c,g} = \frac{\int_{E_{gL}}^{E_{gH}} [\sigma^{Z}_{n,\gamma}(E) + \sigma^{Z}_{n,\alpha}(E) + \cdots] \phi(E)\, dE}{\int_{E_{gL}}^{E_{gH}} \phi(E)\, dE} \quad , \qquad \text{II-24}$$

GROUP FISSION

$$\sigma^{Z}_{f,g} = \frac{\int_{E_{gL}}^{E_{gH}} \sigma^{Z}_{n,f}(E)\,\phi(E)\,dE}{\int_{E_{gL}}^{E_{gH}} \phi(E)\,dE} \quad , \qquad \text{II-25}$$

GROUP SCATTERING (IN OR OUT)

$$\sigma^{Z}_{\{IN\}_{EL},g\to K} = \frac{\int_{E'=E_{gL}}^{E_{gH}}\int_{E=E_{gL}}^{E_{gH}} \sigma^{Z}_{\{n,n'\}_{n,n}}(E\to E')\,\phi(E)\,dE\,dE'}{\int_{E_{gL}}^{E_{gH}} \phi(E)\,dE} \quad , \qquad \text{II-26}$$

where E_{gH} and E_{gL} are the upper and lower limits of group g respectively and $\phi(E)$ is the flux per unit energy interval at E.

Neutrons may be added to a particular group g by a fission source

$$S_{f,g} = \chi_g \sum_{g=1}^{N} (\nu\Sigma_f)_g\,\phi_g \quad , \qquad \text{II-27}$$

where χ_g is the fraction of fission neutrons born into group g such that

$$\chi_g = \int_{E_{gL}}^{E_{gH}} \chi(E)\,dE \quad , \qquad \text{II-28}$$

$$(\nu\Sigma_f)_g = \sum_{Z} N^{Z}(\nu\sigma_f)^{Z}_{g} \quad , \qquad \text{II-29}$$

and

$$(\nu\sigma_f)^{Z}_{g} = \frac{\int_{E_{gL}}^{E_{gH}} \nu^{Z}(E)\,\sigma^{Z}_{n,f}(E)\,\phi(E)\,dE}{\int_{E_{gL}}^{E_{gH}} \phi(E)\,dE} \quad . \qquad \text{II-30}$$

The steady state diffusion equation for the gth energy group, where g represents any group between one and N, can now be written

$$D_g \nabla^2 \phi_g(\bar{r}) - \Sigma_{a,g} \phi_g(\bar{r}) - \left[\sum_{h>g}^{N} \Sigma(g \rightarrow h)\right]\phi_g(\bar{r}) + \sum_{h=1}^{g-1} \Sigma(h \rightarrow g)\phi_h(\bar{r}) + \chi_g \sum_{h=1}^{N} \nu_h \Sigma_{f,h} \phi_h(\bar{r}) = S_g(\bar{r}) \quad . \qquad \text{II-31}$$

In Equation II-31 the inelastic removal cross section from group g appears as the third term of the equation rather than as an implied part of the second term. This equation now represents a set of N coupled partial differential equations independent of time and applicable to one region of a reactor. The right hand side of the equation represents the number of neutrons at energies within group g coming from any extraneous neutron sources.

D. SPACE-INDEPENDENT FORM OF MULTIGROUP EQUATION

For this case, it is assumed that the extrapolation distance at the surface of the reactor is identical for all energy groups and thus is independent of energy. If this is true then all energy groups have the same spatial dependence. The flux is then written as

$$\phi_g(\bar{r}) = \phi_g F(\bar{r}) \qquad \text{II-32}$$

where ϕ_g is the magnitude of the flux in the gth energy group, and the function $F(\bar{r})$ satisfies the equation

$$\nabla^2 F(\bar{r}) + B^2 F(\bar{r}) = 0 \qquad \text{II-33}$$

the factor B^2 is the square of the first eigenvalue of Equation II-33 and is called the buckling.

By substituting Equations II-32 and II-33 into Equation II-31 expanding the resulting equation in a series of eigenfunctions, and applying the orthogonality condition, the result will be a set of N linear algebraic equations in N unknowns $\phi_1, \phi_2, \cdots \phi_N$ written

$$-\left[D_g B^2 + \Sigma_{a,g} + \sum_{h>g}^{N} \Sigma(g \to h)\right] \phi_g + \sum_{h=1}^{g-1} \Sigma(h \to g)\, \phi_h + \chi_g \sum_{h=1}^{N} \nu_h \Sigma_{f,h}\, \phi_h = S_g \quad . \qquad \text{II-34}$$

This set of equations is easily solved by Cramer's Rule.

E. EXPLICIT SPATIAL DEPENDENCE

In order to solve the set of Equations II-31 by numerical methods, the Laplacian operator in the first term must be replaced by its equivalent finite difference approximation. The resulting equation for a typical energy group g in cylindrical geometry is

$$\begin{aligned} & D_g\left[\frac{1}{h^2}\left\{\phi_g(r+1,z,\theta) - 2\phi_g(r,z,\theta) + \phi_g(r-1,z,\theta)\right\}\right. \\ & + \frac{1}{2rh}\left\{\phi_g(r+1,z,\theta) - \phi_g(r-1,z,\theta)\right\} + \frac{1}{h^2}\left\{\phi_g(r,z-1,\theta)\right. \\ & \left. - 2\phi_g(r,z,\theta) + \phi_g(r,z+1,\theta)\right\} + \frac{1}{r^2\gamma^2}\left\{\phi_g(r,z,\theta+1)\right. \\ & \left.\left. - 2\phi_g(r,z,\theta) + \phi_g(r,z,\theta-1)\right\}\right] - \sum_{g-1} a,g\ \phi_g(r,z,\theta) \\ & -\left[\sum_{h>g}^{N} \Sigma(g \to h)\right] \phi_g(r,z,\theta) + \sum_{h=1}^{g-1} \Sigma(h \to g)\, \phi_h(r,z,\theta) \\ & + \chi_g \sum_{h=1}^{N} \nu_h \Sigma_{f,h}\, \phi_h(r,z,\theta) = S_g(r,z,\theta) \quad . \end{aligned} \qquad \text{II-35}$$

Appendix A contains the details of the derivation of the finite difference approximation and the definition of the symbols used in Equation II-35.

Throughout this investigation, circular summetry was imposed. Thus the third term in Equation II-35 is zero. For mesh points not on the longitudinal axis (See Appendix A for finite difference operator used on this axis.), Equation II-35 becomes

$$D_g\left[\frac{1}{h^2}\{\phi_g(r+1,z)-2\phi_g(r,z)+\phi_g(r-1,z)\}\right.$$
$$+\frac{1}{2rh}\{\phi_g(r+1,z)-\phi_g(r-1,z)\}+\frac{1}{h^2}\{\phi_g(r,z+1)$$
$$\left.-2\phi_g(r,z)+\phi_g(r,z-1)\}\right]-\Sigma_{a,g}\phi_g(r,z) \qquad \text{II-36}$$
$$-\left[\sum_{h>g}^{N}\Sigma(g\to h)\right]\phi_g(r,z)+\sum_{h=1}^{g-1}\Sigma(h\to g)\phi_h(r,z)$$
$$+\chi_g\sum_{h=1}^{N}\nu_h\Sigma_{f,h}\phi_h(r,z)=S_g(r,z) \quad .$$

F. MATRIX FORM OF MULTIGROUP EQUATIONS

For the purpose of numerical analysis and computational ease, Equation II-36 is reduced to the matrix form. In Appendix B, this equation is first expanded to fit the available sixteen group data, then reduced to a form more suitable for matrix notation. Finally, it is reduced to the form

$$A1\,\phi_g(r-1,z)+A2\,\phi_g(r,z)+A3\,\phi_g(r+1,z)$$
$$+A4\phi_g(r,z-1)+A5\,\phi_g(r,z+1)=S_g(r,z) \qquad \text{B-4}$$
$$+AS_1\,\phi_1(r,z)+\cdots+AS_{g-1}\,\phi_{g-1}(r,z)$$
$$+AF_g\,\phi_g(r,z)+\cdots+AF_{18}\,\phi_{16}(r,z)$$

To reduce this set of multigroup equations to matrix form, the spatial dependence is considered first.

The flux vector $\overline{\phi}_g$ of group g is defined as

$$\phi_g = \begin{bmatrix} \phi_g(1,1) \\ \vdots \\ \phi_g(L,1) \\ \vdots \\ \phi_g(1,M) \\ \vdots \\ \phi_g(L,M) \end{bmatrix} , \qquad \text{II-37}$$

where $\phi_g(L,2\rightarrow M)$ and $\phi_g(2\rightarrow L,M)$ are on the extrapolated boundary and are set equal to zero. The fluxes $\phi(1,X)$ and $\phi(X,1)$ are set equal to the fluxes $\phi(3,X)$ and $\phi(X,3)$ respectively in order to satisfy the normal derivative boundary condition. See Appendix C for mesh point numbering scheme.

The matrix of coefficients $\bar{A}g$ is written

$$\bar{A}_g = \begin{bmatrix} O\ A4\ O\cdots O & A1\ A2\ A3\ O\cdots O & O\ A5\ O\cdots O\cdots O \\ O\ \ O\ A4\cdots O & O\ A1\ A2\ A3\cdots O & O\ \ O\ A5\ O\cdots O \\ \vdots & & \\ O\cdots\cdots O & O\cdots\cdots A4 & O\cdots A1\cdots A5 \end{bmatrix} , \qquad \text{II-38}$$

The terms on the right hand side of equation B-4 are called the source terms and $\overline{W}g$ is the source vector for group g. It can be

represented in matrix form by

$$\overline{W}_g = \sum_{h=1}^{g-1} \overline{AS}_h \, \overline{\phi}_h + \sum_{h=g+1}^{18} \overline{AF}_h \, \overline{\phi}_h + \overline{S}_g \, \overline{\phi}_h \qquad \text{II-39}$$

where the flux vector $\overline{\phi}_h$ is given as

$$\overline{\phi}_h = \begin{bmatrix} \phi_h(1,1) \\ \phi_h(2,1) \\ \vdots \\ \phi_h(L,1) \\ \vdots \\ \phi_h(L,M) \end{bmatrix}, \qquad \text{II-40}$$

the scattering matrix $\overline{AS}_h$

$$\overline{AS}_h = \begin{bmatrix} 0 \cdots 0 & 0 \; AS_h \; 0 \cdots 0 & 0 \cdots 0 \\ 0 \cdots 0 & 0 \; 0 \; AS_h \cdots 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots \\ 0 \cdots 0 & 0 \cdots 0 & 0 \cdots AS_h \end{bmatrix} \qquad \text{II-41}$$

the fission matrix $\overline{AF}_h$

$$\overline{AF}_h = \begin{bmatrix} 0 \cdots 0 & 0 \; AF_h \; 0 \cdots 0 & 0 \cdots 0 \\ 0 \cdots 0 & 0 \; 0 \; AF_h \cdots 0 & 0 \cdots 0 \\ \vdots & \vdots & \vdots \\ 0 \cdots 0 & 0 \cdots 0 & 0 \cdots AF_1 \end{bmatrix} \qquad \text{II-42}$$

and the extraneous source matrix $\bar{S}g$

$$\bar{S}_g = \begin{bmatrix} 0 \cdots 0 & 0 \; S_g \; 0 \cdots 0 & 0 \cdots 0 \\ 0 \cdots 0 & 0 \; 0 \; S_g \cdots 0 & 0 \cdots 0 \\ \vdots & & \\ 0 \cdots 0 & 0 \cdots 0 & 0 \cdots S_g \end{bmatrix} \quad \text{II-43}$$

The result is then a set of equations, one for each spatial mesh point for the energy group g. These may be written as

$$\bar{A}_g \, \bar{\phi}_g = \bar{W}_g \quad \text{II-44}$$

Next the equations are cast according to the group dependence. Define $\bar{\phi}$ as

$$\bar{\phi} = \begin{bmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \\ \vdots \\ \bar{\phi}_G \end{bmatrix}, \quad \text{II-45}$$

$\bar{A}$ as

$$\bar{A} = \begin{bmatrix} A_1 \; 0 \cdots 0 \\ 0 \; A_2 \cdots 0 \\ \\ 0 \cdots A_g \end{bmatrix}, \quad \text{II-46}$$

and $\overline{W}$ as

$$\overline{W} = \begin{bmatrix} \overline{W}_1 \\ \vdots \\ \overline{W}_N \end{bmatrix} \qquad \text{II-47}$$

The entire set of equations then takes the form

$$\overline{A}\,\overline{\phi} = \overline{W} \qquad \text{II-48}$$

G. NUMERICAL SOLUTION OF THE MULTIGROUP EQUATIONS

The basic equation II-48 can be solved by various direct or iterative methods. Direct methods generally are based on either Gaussian elimination with pivoting, or on triangular decomposition of the matrix of coefficients. Iterative methods include the Jacobi method, unextrapolated Liebmann or Gauss-Seidel method, various forms of the extrapolated Liebmann or successive orerrelaxation method and others.

In this study, up to 25,000 algebraic equations were expected as a result of a sixteen group problem in two dimensions with 1600 mesh points. For this large number of equations, iterative methods are generally used rather than direct methods [8, 9]. Smith [9] states that one reason for this is that the iterative methods are more efficient in that they take advantage of the sparse matrix of coefficients with its numerous zero elements.

In this study, the point successive overrelaxation method was used (See Appendix D).

First the reactor materials and energy group structure are chosen. Next the cross section data is computed or as was the case in this study, taken from some reliable source [7] and used to compute the appropriate coefficients. Then initial estimates of the group fluxes designated $\overline{\phi}_g^\circ$ at all mesh points in the reactor except those lying on the extrapolated boundaries are made. These values are then used along with their respective coefficients to compute the initial source vectors $\overline{W}_g^\circ$. The flux in group one at all mesh points in the reactor is then corrected by solving the set of equations

$$\overline{A}_1 \, \overline{\phi}_1^\circ = \overline{W}_1^\circ \qquad \text{II-49}$$

Using the point successive overrelaxation method, this is done by solving the first of Equations II-49 for the central mesh point, i.e., $\phi_1(2,2)$ and then solving the remaining equations point-by-point, row-by-row, at all other points no on the extrapolated boundary. The solution is denoted $\overline{\phi}_1^1$. The remaining group equations are then similarly solved yielding the vector $\overline{\phi}^1$ where

$$\overline{\phi}^1 = \begin{bmatrix} \overline{\phi}_1^1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \overline{\phi}_N^1 \end{bmatrix} , \qquad \text{II-50}$$

which represents the first iteration.

This vector is then rescaled such that

$$(S_1 \overline{\phi}^1, S_1 \overline{\phi}^1) = (\overline{\phi}^\circ, \overline{\phi}^\circ) , \qquad \text{II-51}$$

which indicates that the flux vector on the zeroth iteration integrated over all spatial dimensions and all energy groups is equal to the flux vector after the first iteration integrated over all spatial dimensions and all energy groups multiplied by a scale factor [10]. When the process is carried out, it can be seen that after a few iterations the scale factor will approach a constant value. The reason for this as described by Lamarsh [11] is that each iteration is actually equivalent to one cycle of the chain reaction. The zeroth iteration of the fluxes $\overline{\phi}^{\circ}$ then can be thought of as a set of initial conditions or the flux state at time t = 0. Eventually the flux will approach the funadmental eigenfunction as the higher harmonics die out. The fundamental will then vary with the number of iterations (time) increasing or decreasing depending on whether the system is subcritical or critical. The quantity $\mathfrak{Z}$ can clearly be considered a measure of the criticality of the system. If $\mathfrak{Z}$ is less than unity then the group fluxes throughout the system are decreasing with each iteration and the system is subcritical and if $\mathfrak{Z}$ is greater than unity then the fluxes are increasing and the system is supercritical. $\mathfrak{Z}$ is in fact the inverse of the effective multiplication factor K_{eff}.

The iteration process described above is referred to as the inner iteration or flux iteration. If a critical assembly is desired, then an outer iteration must be performed wherein adjustments are made to the composition or size of the reactor in order to make the effective multiplication factor converge to unity.

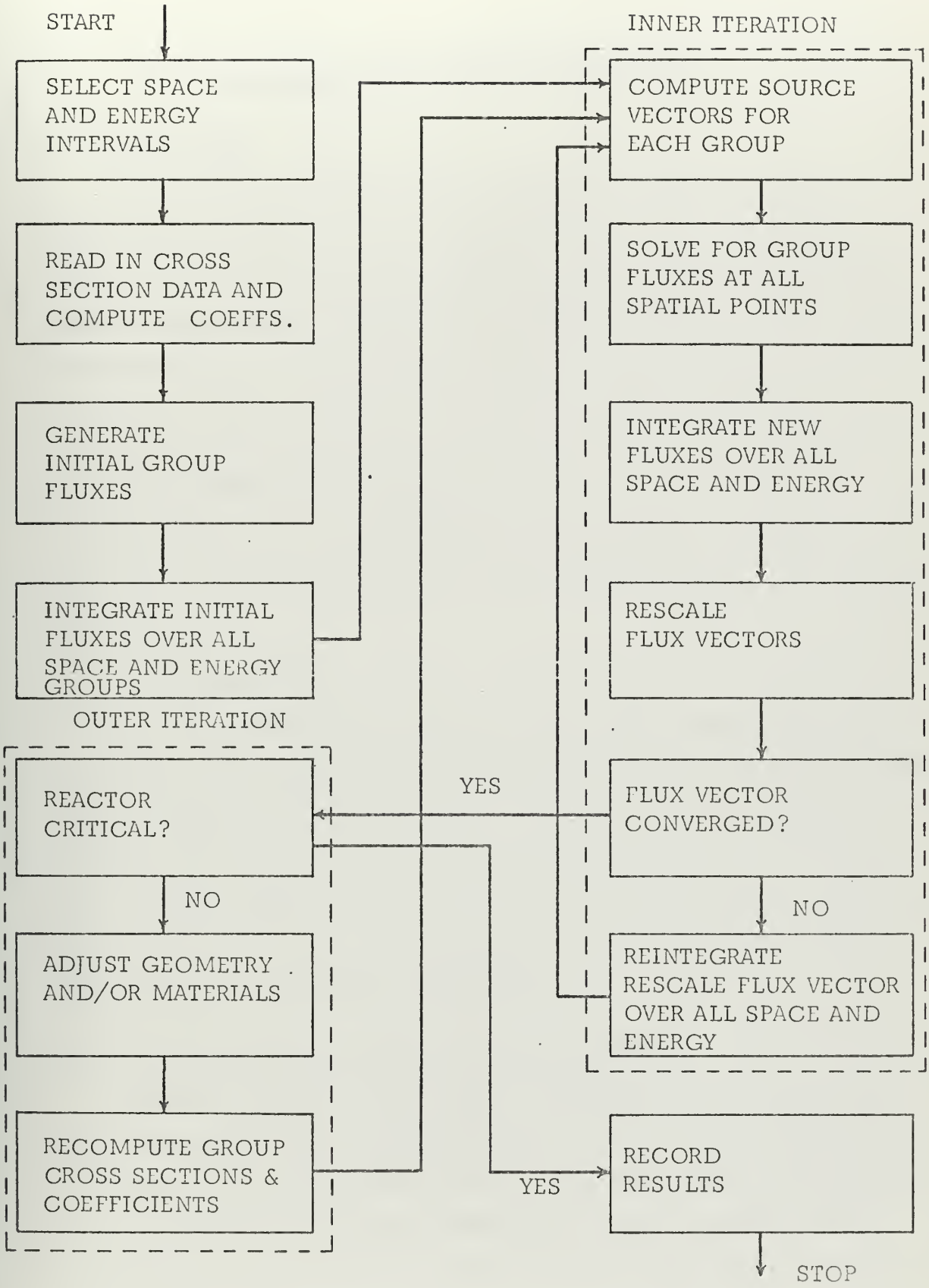
Convergence of the flux vectors on the inner or flux iterations is determined by taking the absolute value of the difference between the flux at all mesh points and in all energy groups and comparing it with some desired value ϵ_1, such that

$$\left| \phi^{N-1}(E,r,z) - \phi^{N}(E,r,z) \right|_{\substack{ALL \\ E,r,z}} < \epsilon_1 \qquad \text{II-52}$$

Convergence of the outer or criticality iteration is determined by subtracting the scale factor ζ from unity so that

$$\left| 1.0 - \zeta \right| \leq \epsilon_2 \quad , \qquad \text{II-53}$$

where ϵ_2 is an arbitrary convergence criterion.



PROGRAM FLOW CHART

III. DESCRIPTION OF COMPUTER PROGRAM

A. INPUT DATA

In this section the input data is read in via punched cards as illustrated in Appendix E and printed in the output for verification. The input data along with their symbolic program designators include the following: the energy dependent source SS (I); the maximum values of the initial flux estimate S(I); the energy intervals DELE(I); the number of atoms per cubic centimeter H(I) of each element or isotope; the microscopic cross sections for each element or isotope for inelastic scattering SIGIN(I,J,K), transport SIGTR(I,J), fission SIGF(I,J), elastic removal SIGER(I,J), and capture SIGC(I,J); the normalized neutron fission spectrum CHI(I); and the average number of neutrons per fission NU(I,J).

B. OUTER ITERATION

If a critical composition is the desired result, the program returns to this section after the iteration has converged (See SAMPLE COMPUTER PROGRAM page 119). The iteration on composition then begins by adding to the volume of the reactor an incremental volume of uranium 235. An equal volume of uranium 238 is subtracted and new values for N^{25} H(1) and N^{28} H(2) are computed. On the zeroth outer iteration this section of the program is by-passed.

C. COMPUTATION OF MATRIX OF COEFFICIENTS

Input data is used in this section to compute the elements of the coefficient matrix, the scattering matrix, and the fission source matrix. In the first part of this section, the various microscopic cross sections for each reactor component are multiplied by their respective atomic abundances N^x. The values for each particular reaction (for example, capture or inelastic scatter from g to g + 1) are then summed. This

is done for each energy group. The summations are then combined with other input data and the coefficients are computed for each group equation.

The equation denoted by statement number 17 in the sample program computes the diffusion coefficients for each group based on the simple diffusion theory approximation. The absorption coefficients for each group are computed in the equation denoted by statement number 175 by summing the contributions from all the absorption or loss reactions. The equation denoted by statement number 18 computes the fission and down-scattering coefficients from all higher energy groups. This is accomplished by starting at the highest energy group in which the reactions originate and computing its contribution to all lower energy groups at the end of the slowing-down process. The fission source coefficients for a particular group and all lower energy groups are computed by the equation given in statement number 21. The above coefficients are completely defined in Section II and Appendix B.

D. INITIAL FLUX ESTIMATE AND BOUNDARY CONDITIONS

In the cylindrical geometry the flux shape is initially estimated as the product of the relative group magnitude factor S(I), a zero order Bessel function, and a cosine function. The factors S(I) were determined for this study as suggested by Clark and Hansen [10].

The flux on the exterior boundaries is set equal to zero. At the interior boundaries the flux at the first mesh points on either side of the longitudinal and mid-plane axes are set equal, indicating that the flux has its maximum value along these axes. At the end of this section the initial flux estimate is printed out for verification and comparison with the final flux values.

The program is designed to by-pass this section on all outer iterations.

E. INITIAL FISSION SOURCE INTEGRATION

The initial flux estimate is used in this section to compute the fission source vector for the zeroth inner iteration. The program performs the following triple integration:

$$\int_0^R \int_0^Z \int_0^E \left\{ \left[\phi(E,r,z) \right]^2 \chi(E) \sum_i^C \nu_i(E) \Sigma_{f,i}(E) \right\} dE\, dr\, dz \qquad \text{III-1}$$

by dividing the energy spectrum into discrete intervals DELE(I) corresponding to the energy groups (for this study the upper boundary of the highest energy level was taken as 10 Mev for use in this calculation) and dividing the area into finite intervals corresponding to the mesh spacing GS. A summation process is then carried out.

F. COMPUTATION OF SOURCE VECTORS

The components of the vector $\overline{W}$ for each inner iteration are computed by summing the products of the fission source or the combined fission source plus down-scattering source coefficients and the scaled flux values from the previous iteration. On the zeroth iteration the initial flux estimate is used with a scale factor of unity. The resulting numbers SUMR(I,R,Z) and SUML(I,R,Z) are the space dependent components of each group source vector $\overline{W}g$.

G. SOLUTION OF THE DIFFUSION EQUATION

This section consists of one outer energy group loop with several space loops within it. Each group equation beginning with the highest group is solved at all spatial points starting at the longitudinal axis r = 0 and proceeding column by column to within one mesh point of the extrapolated boundaries r = R-1 and z = Z-1.

The first space loop will solve the diffusion equation at the geometric center of the reactor, i.e., point (2,2) (See Appendix C, Figure C-1.) if an extraneous point source is located in this position. This particular configuration was used in this study; however, extraneous sources with other spatial distributions could also be used with the appropriate changes to this section of the program.

The second space loop involves the solution of the diffusion equation at all points on the longitudinal axis r = 0 but not occupied by an extraneous source. A special finite difference approximation for points on this axis is necessary due to a singularity caused by the radius being equal to zero (See Appendix A, page 66).

The last spatial loop solves the diffusion equation at all remaining mesh points with the exception of those located on the exterior boundaries.

Within each of these spatial loops, the flux value at each mesh point from the previous iteration is unscaled (TPHI1) and compared with the flux value at each mesh point in the present iteration to determine convergence as defined in Section II, page 27. The largest absolute value of these differences,

$$TEST2 = DABS(TPHI2 - TPHI1), \qquad \text{III-2}$$

is stored as the quantity SAVE and may be printed with the output as desired. The scaled flux values of the previous iteration SPHI1, SPHI2, SPHI3 are used in the successive point overrelaxation scheme for solving the group equations at each mesh point.

H. NTH FISSION SOURCE INTEGRATION (UNSCALED FLUX)

In this section the triple integration process, Equation III-1, is performed with the new but as yet unscaled flux values.

I. COMPUTE SCALE FACTOR AND RENORMALIZE FLUX

Once the fission source integration has been computed for the N-1 iteration FLUXI(NN-1) using the scaled or renormalized flux values and for the Nth iteration FLUXI(NN) using unscaled flux values, the scale may be determined from

$$SCALF = DSQRT(FLUXI(NN-1)/FLUXI(NN)) \qquad III-2$$

The flux values at all mesh points and in all energy groups are then renormalized by multiplying by this scale factor. The scale factor SCALF is the principal eigenvalue of the system and its reciprocal is the effective multiplication constant SKEFF as explained in Section II, page .

J. NTH FISSION SOURCE INTEGRATION (SCALED FLUX)

At this point the integration process of Section III(E) is performed using the renormalized flux. On the subsequent iteration this will become the quantity FLUXI(NN-1) in the numerator of Equation III-2.

K. END OF INNER ITERATION

This statement (1801) is the controlling statement for the inner iteration. It can make the program exit to either the output section or the outer iteration loop depending upon whether or not a critical reactor is desired. The controlling factor may be either a specific number of inner iterations if experience has shown that the convergence criterion as defined in Section II, page 27 will be satisfied or another possibility would be to stop the inner iteration once the convergence criterion is met regardless of the number of inner iterations. The second method might be preferable once it had been determined that convergence to a reasonable criterion, i.e., ϵ_1, would not take an excessive amount of computer time.

L. OUTPUT SECTION

The output is very flexible and may be easily changed to fit a particular problem or set of problems. Typical output data includes the following: scale factor, effective multiplication constant, flux values at various mesh points and various inner or outer iterations, convergence criterion SAVE (ϵ_1) and overrelaxation factor w. In addition, graphical output in the form of graphs of flux shape compared with the cosine function and zero order Bessel function at various locations may be plotted and the convergence criterion SAVE versus iterations. Many other possibilities exist.

M. START NTH OUTER ITERATION AND TEST FOR CRITICALITY

In this section the scale factor is checked out for criticality by comparison with an arbitrarily selected value (.0005 was chosen for this study.) such that

$$|1.0 - \text{SCALF}| \leq .0005 \qquad \text{III-3}$$

If the above condition is not satisfied, then another outer iteration is made. The number of outer iterations may also be arbitrarily limited in this section. Once this limit, for example, ten outer iterations, is reached, the program will stop even though criticality has not been reached.

IV. RESULTS AND PROGRAM VERIFICATION

If in the paragraphs that follow modifications to the basic core described below have been made, these modifications will be so stated. The results described and tabulated in this section were based on a typical homogeneous fast reactor core composed of the following materials with their respective volume fractions:

U^{235} - 2%
U^{238} - 30%
Na - 33%
Fe - 27%
Pu^{239} - 8%

This composition was chosen to coincide approximately with the RAPSODIE reactor core in reference [12]. The core dimensions were 36 centimeters in radius and 72 centimeters in height. Mesh spacing was two centimeters in both the radial and axial directions. Three energy groups were used with the cross section data taken from ANL 5800 [7].

A. DETERMINATION OF OPTIMUM RELAXATION FACTOR

The experimental method used for determining w_{opt} consisted of two basic steps. First, an analytical estimation was obtained based on formulation developed by Frankel [13] and Starling [14].

Frankel [13] derived the following equations based on the solution to Laplace's equation in a bounded region R in rectangular coordinates

$$w_{opt} = 4\alpha \qquad \text{IV-1}$$

where α is the smaller root of

$$\alpha^2 t^2 - 4\alpha + 1 = 0 \quad , \qquad \text{IV-2}$$

and t is given by

$$t = \cos\frac{\pi}{p} + \cos\frac{\pi}{q} \qquad \text{IV-3}$$

where p and q are the number of mesh points in the x and y directions. Smith [9] used this same method to solve the similar Poisson's equation in rectangular coordinates in a bounded region R. Starling [14] has developed an equation for t applicable to Laplace's equation in cylindrical coordinates and given by

$$t = \cos\frac{\pi h}{Z} + J_o\left(\frac{3.8317h}{R+h}\right) \quad , \qquad \text{IV-4}$$

where Z and R are the height and radius of the cylinder respectively and h is the mesh spacing.

The table below gives the results found for these analytical solutions for w_{opt} using a symmetric cylindrical section 36 centimeters in radius and 36 centimeters in half-height

METHOD FOR FINDING "t"	w_{opt}	REMARKS
Eq. IV-4	1.7888	Z = 72 cm. (full height)
Eq. IV-4	1.7259	Z = 36 cm. (half height)
Eq. IV-3	1.7294	p = q = 20 (no. mesh pts. + 1)
Eq. IV-3	1.704	p = q = 18 (no. mesh pts.)

The second step in the experimental process was to try these values in the actual program and graph the convergence criterion versus the number of inner iterations for these values of w_{opt} along with other reasonable estimates to determine the best one. Figure IV-1 shows the

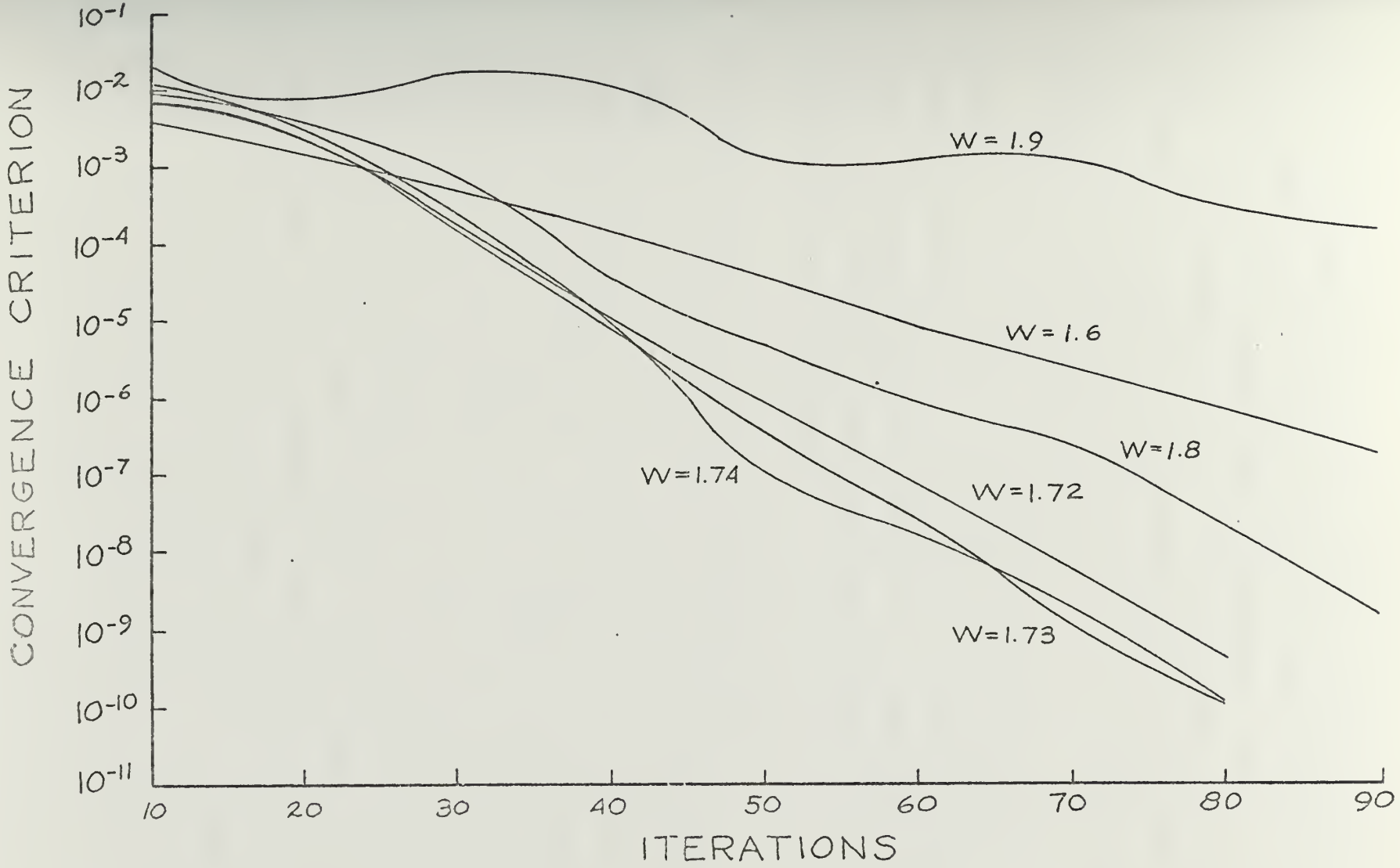

CONVERGENCE CRITERION
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
10-11
10
20
30
40
50
60
70
80
90
ITERATIONS
W = 1.9
W = 1.6
W=1.8
W=1.74
W=1.72
W=1.73

Figure IV-1

data which resulted from this study. It can be seen that for the values of w_{opt} equal to 1.72, 1.73 and 1.74, a convergence criterion value of less than 10^{-6} is satisfied within fifty iterations. Based on this data, it was concluded that Equation IV-4 with Z equal to one-half the height of the cylinder, due to symmetry, gave the best value of w_{opt} for this program. The program was modified to compute w_{opt} automatically based on Equation IV-4.

The above results were reasonable in view of the fact that Equation IV-4 was developed for use with Laplace's equation in cylindrical coordinates. The multigroup neutron diffusion equation used in this study in cylindrical geometry is of a similar form. One of the major differences is that the dependent variable ϕ is a function of both space and energy.

B. FLUX SHAPES FOR A TYPICAL NONCRITICAL CORE

For the bare reactor described at the beginning of this section, the initial flux shape was estimated to be the product of the relative group weighting factor S(I), a cosine function and a zero order ordinary Bessel function of the first kind. This corresponds to the solution of Laplace's equation in a finite cylinder. Figure IV-2 shows the final flux shape in the axial direction at the center of the reactor after 260 inner iterations with a convergence criterion of less than 10^{-10} compared with the cosine shape at the same location at zero iterations. The magnitudes were normalized to unity so that the shapes could be easily compared. It was found that the deviations were in the order of 1 to 15 per cent with the per cent deviation increasing from the center outward. All three energy groups showed exactly the same normalized shapes after an equal number of iterations. The magnitudes of the group fluxes are unknown since they depend on the power level at which the reactor is being operated.

AXIAL FLUX SHAPE

IN A TYPICAL SUBCRITICAL CORE

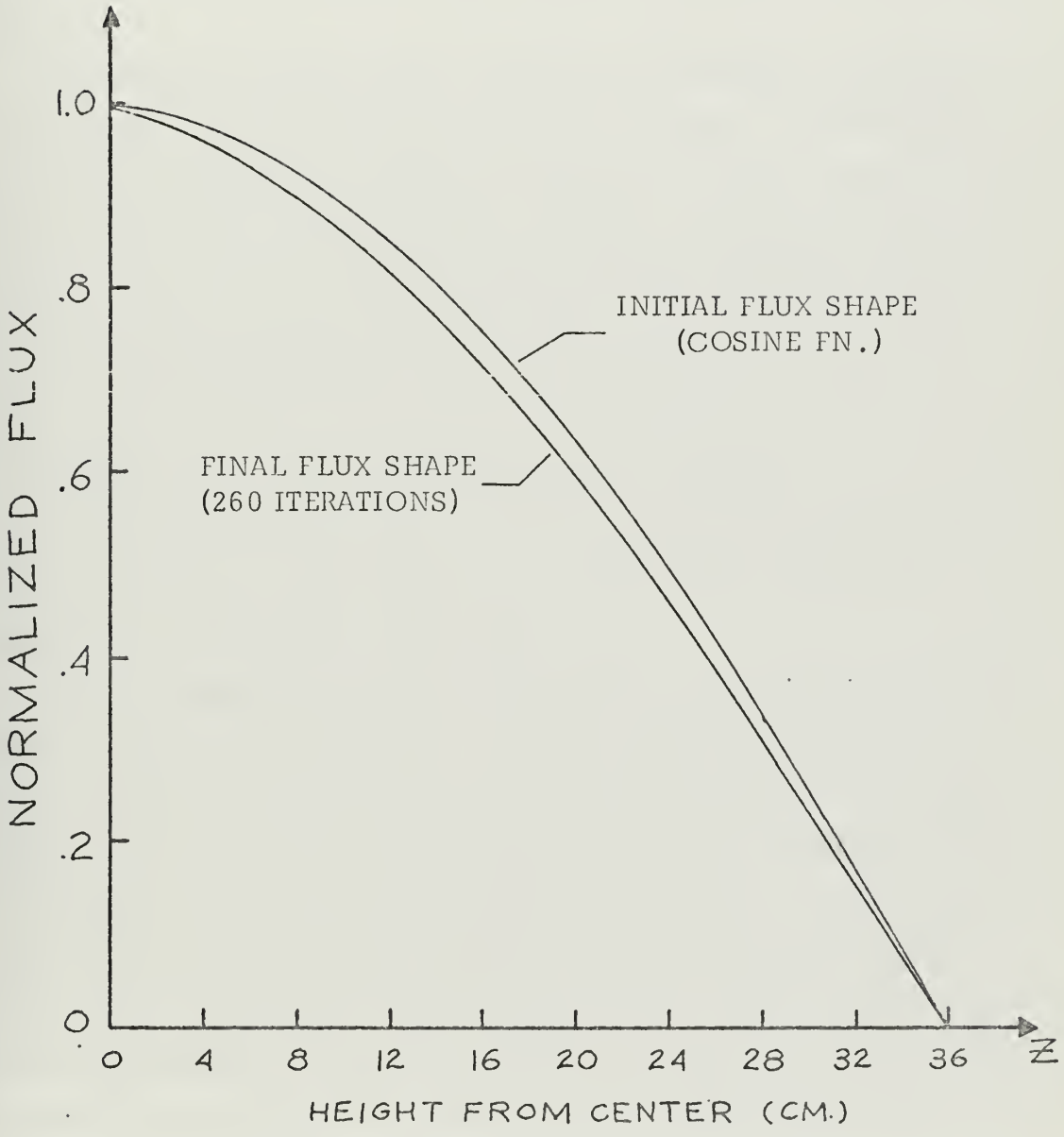


Figure IV-2

The same comparison has been made in Figure IV-3 between the flux in the radial direction and the assumed initial Bessel function. A very slight deviation in shapes was noted here also indicating the close similarity between the diffusion equation and Laplace's equation.

Examination of the data revealed that the final flux shapes were in effect established completely to seven decimal places after some 60 inner iterations at which time the convergence criterion SAVE was less than 10^{-7}.

The program was run using a constant as the initial flux shape estimate rather than the cosine and Bessel function. The resulting final shapes for all three groups were exactly the same as those in Figures IV-2 and IV-3 after 90 inner iterations. Convergence was somewhat slower using this initial estimate. For example, after 50 iterations, SAVE was equal to 3.2×10^{-5} versus 3.7×10^{-7} for an initial estimate based on the cosine, Bessel function product.

C. PROGRAM TEST WITH EXTRANEOUS POINT SOURCE

The program was run with a simulated extraneous Pu-Be point neutron source at the geometric center of the reactor. Two core compositions were used in these tests.

The first core was made up of the typical composition described at the beginning of this section. The flux shapes in the radial and axial directions for the three groups were plotted on the same graph with the Bessel functions and cosine respectively. Figures IV-4 and IV-5 show the results of these comparisons after 90 inner iterations. Group one showed the largest deviation with the most pronounced effect nearest the source. This result was due to the normalized source neutron contribution to group one. Eighty-seven per cent of the neutrons from the Pu-Be source are born with energies lying in group one (1.35 Mev $\rightarrow \infty$) as defined in ANL 5800 [7].

RADIAL FLUX SHAPE

IN A TYPICAL SUBCRITICAL CORE

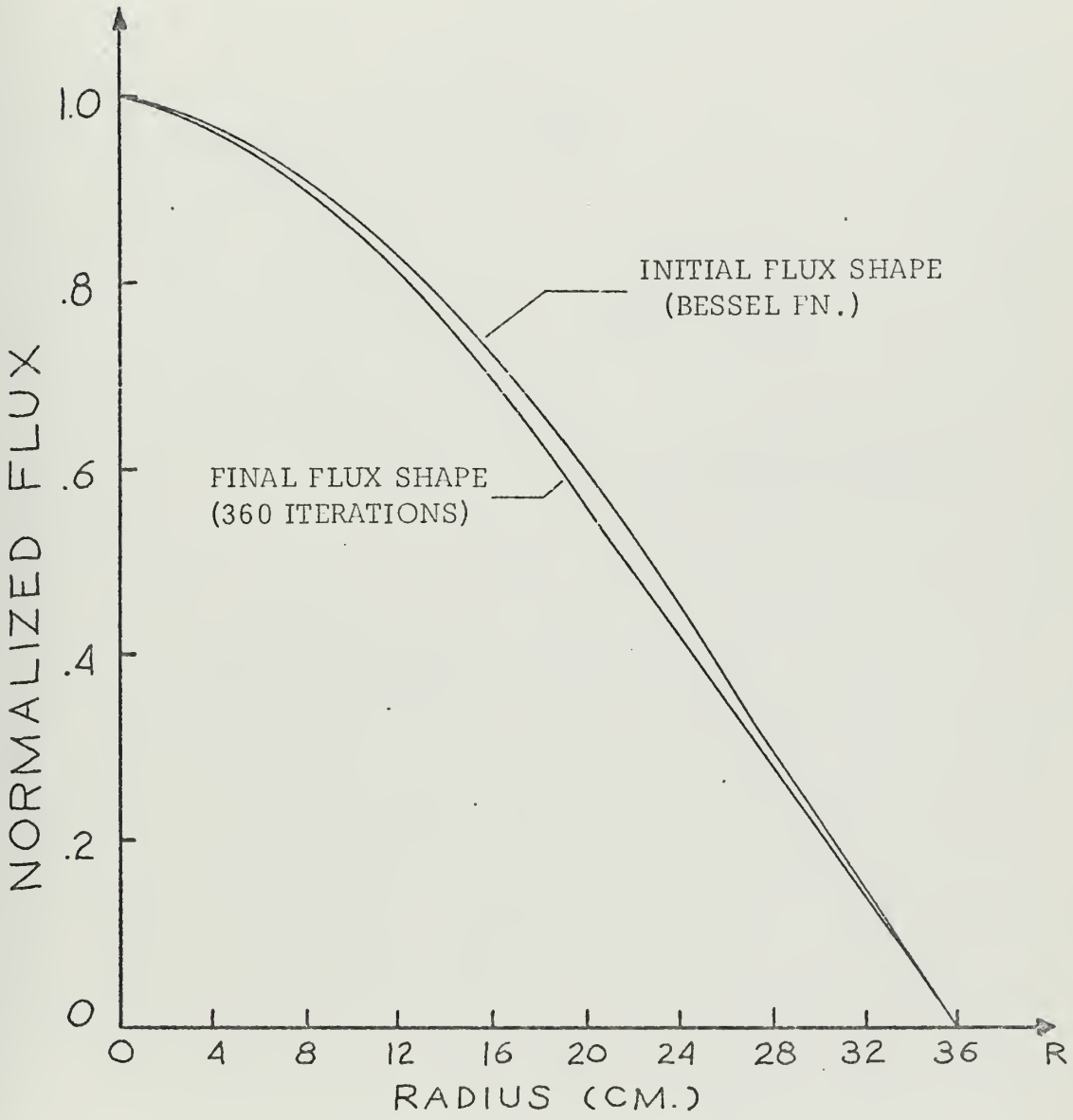


Figure IV-3

RADIAL FLUX SHAPES
IN A TYPICAL SUBCRITICAL CORE
WITH A CENTRAL POINT SOURCE

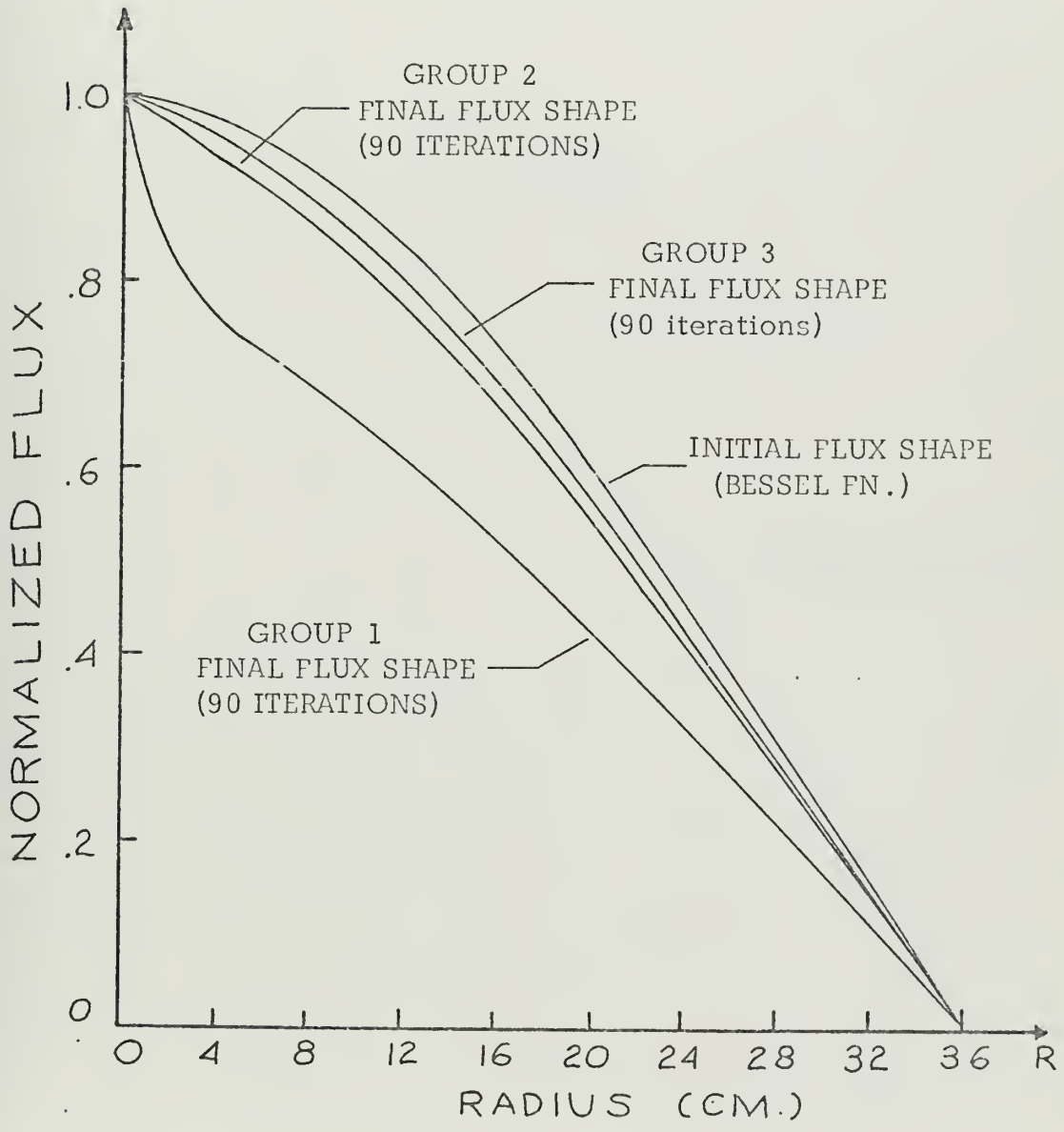


Figure IV-4

AXIAL FLUX SHAPES

IN A TYPICAL SUBCRITICAL CORE

WITH A CENTRAL POINT SOURCE

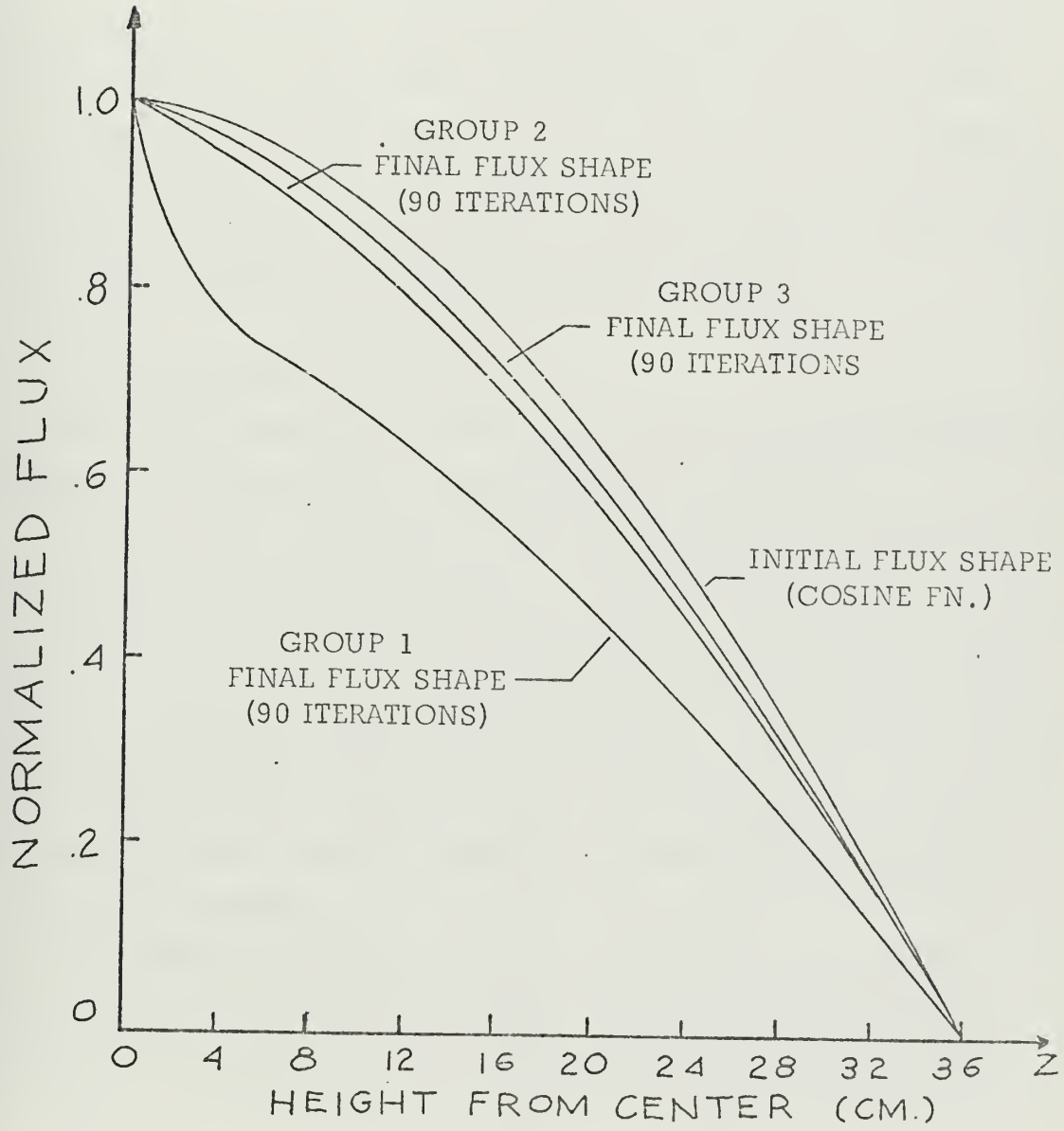


Figure IV-5

The second core was a solid iron cylinder with the same dimensions as the typical fast reactor core. The Pu-Be source was located at the center of the cylinder. The radial flux shapes for groups one and two were plotted in Figure IV-6 and the axial flux shapes for these two groups were plotted in Figure IV-7. It was noted that for these two groups the flux decayed very rapidly as the distance from the source increased; therefore, the ordinates of Figures IV-6 and IV-7 are logarithmic. Figures IV-8 and IV-9 show the radial and axial flux for group three. In these graphs the flux shapes are compared with the cosine and Bessel functions after 90 inner iterations.

The three group flux spectrum at the center of the two assemblies are shown in Figure IV-10. A relatively flat spectrum was observed for the core containing the fissile material while the solid iron core showed large relative differences among the three group fluxes.

D. CRITICALITY TEST

Test of the program up to this point included only the inner iteration. Criticality was achieved by two methods of outer iteration.

The first method consisted of performing sufficient inner iterations for convergence. Seventy iterations were used resulting in an ϵ_1 of less than 10^{-9} for a mesh size of two centimeters. An outer iteration was then carried out wherein .5% by volume of U^{235} was added before each outer iteration as described in Section III-B. This cycle was then repeated until the criticality criterion ($|1.0 - SKEFF| < .0005$) was satisfied. The optimum overrelaxation factor remains the same for each outer iteration.

Four different mesh spacings were used in this section of the program analysis to determine the effect, if any, mesh size had on criticality and flux shapes. Table IV-1 contains the data resulting from this study. The central flux shapes in the radial and axial directions at criticality for all mesh sizes tried were nearly coincident

RADIAL FLUX SHAPE

IN AN IRON CYLINDER

WITH A CENTRAL POINT SOURCE

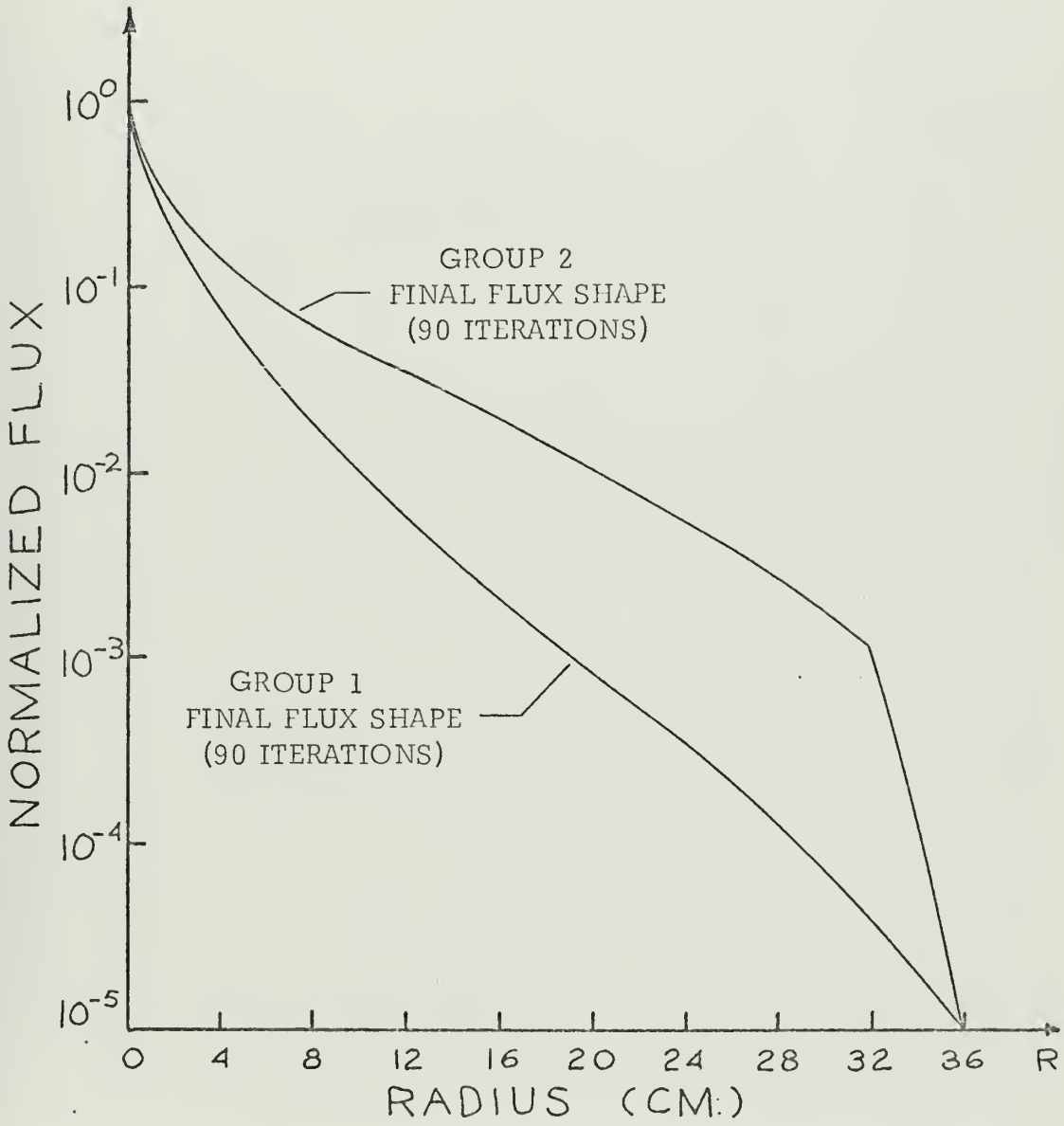


Figure IV-6

AXIAL FLUX SHAPE
IN AN IRON CYLINDER
WITH A CENTRAL POINT SOURCE

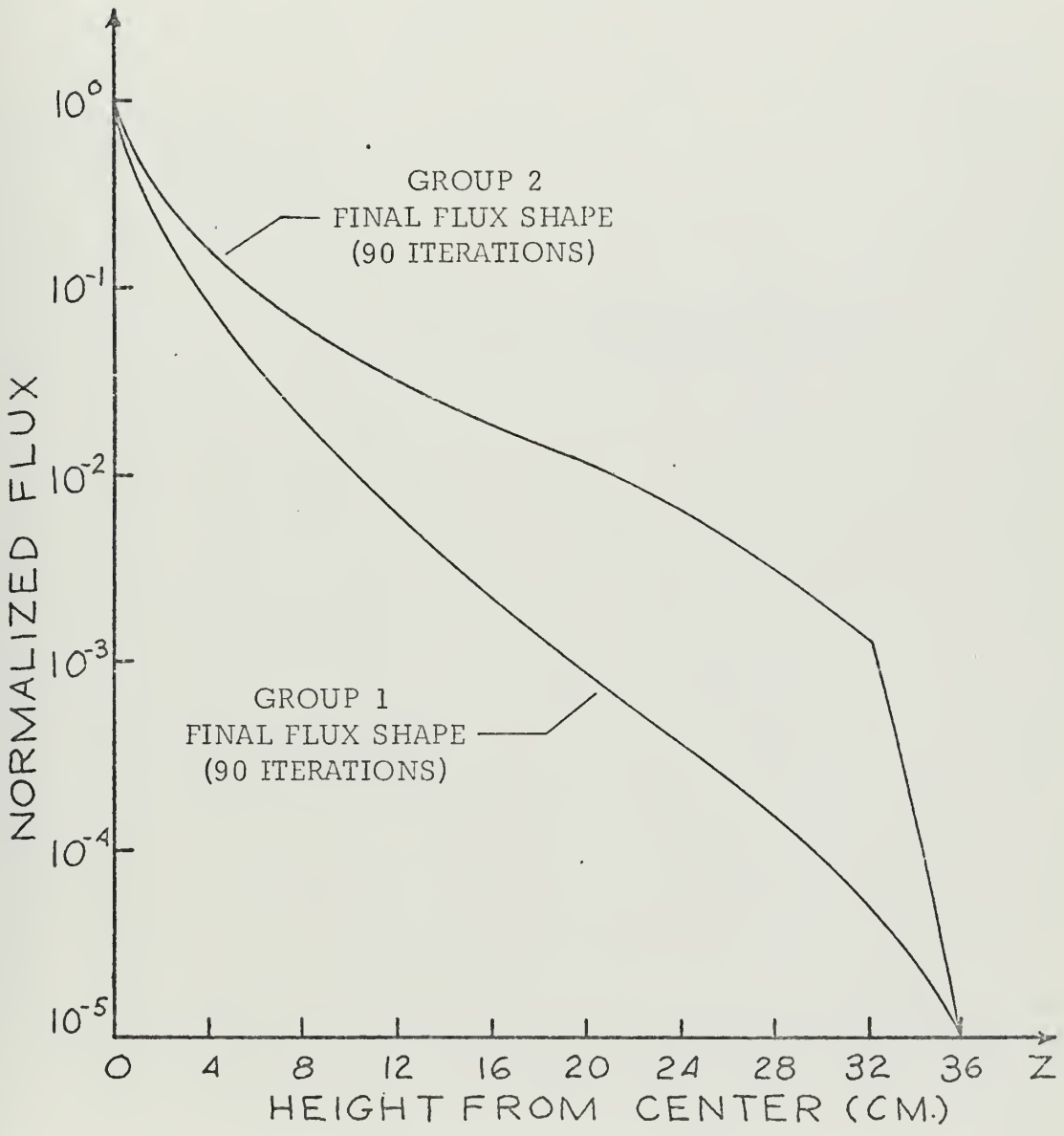


Figure IV-7

RADIAL FLUX SHAPE

IN AN IRON CYLINDER

WITH A CENTRAL POINT SOURCE

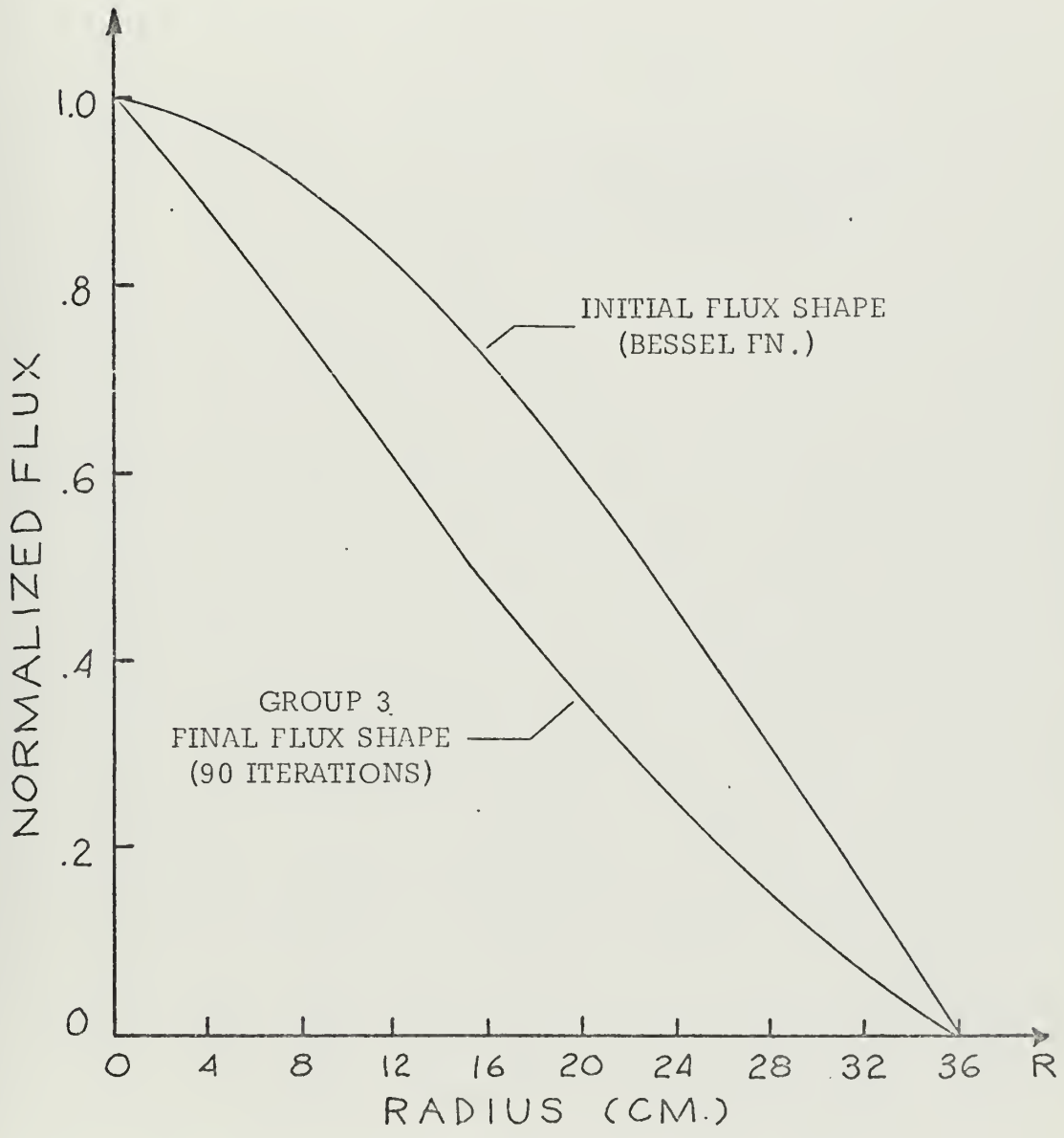


Figure IV-8

AXIAL FLUX SHAPE

IN AN IRON CYLINDER

WITH A CENTRAL POINT SOURCE

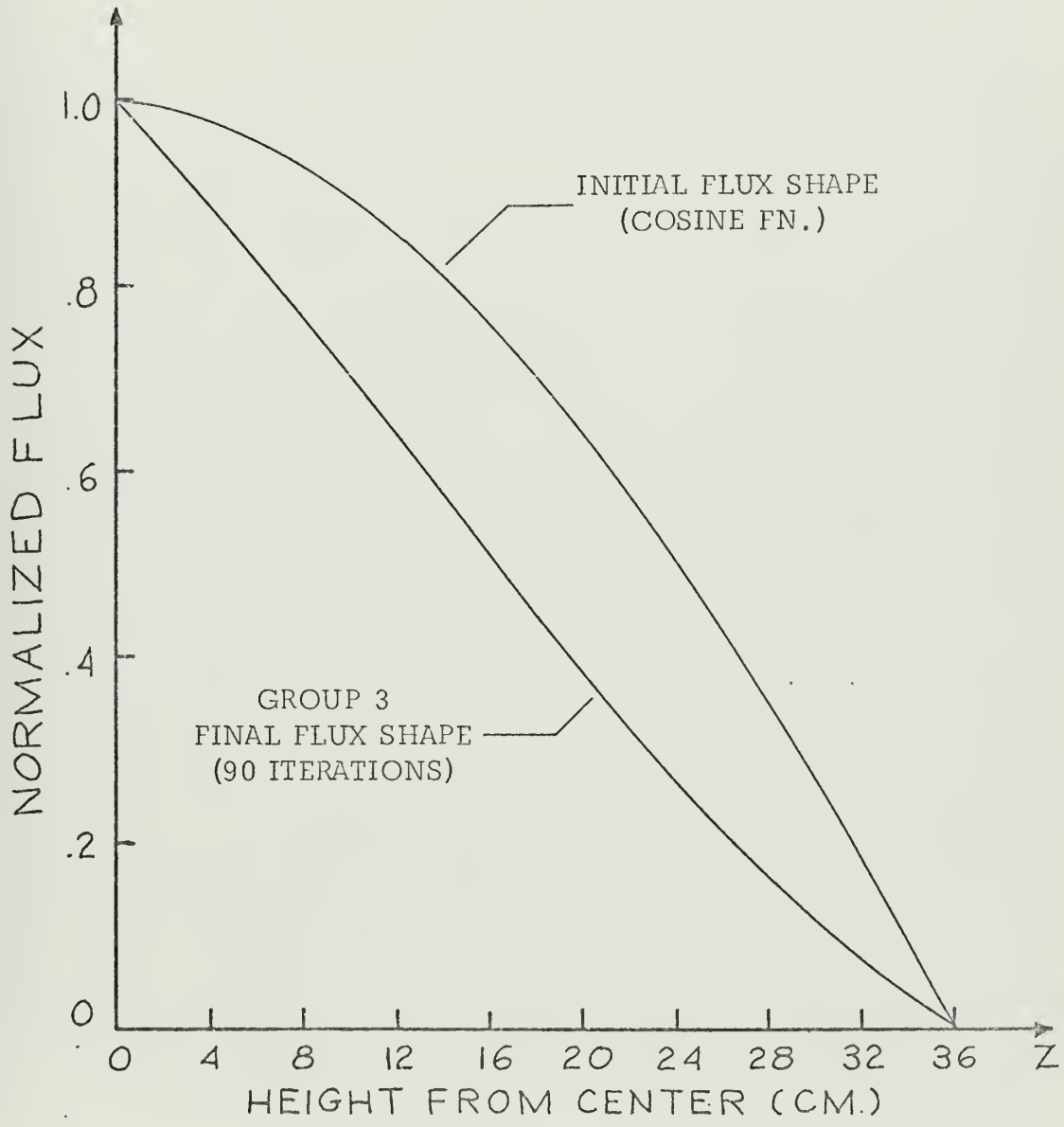


Figure IV-9

FLUX SPECTRUM

AT CORE CENTER

WITH A CENTRAL POINT SOURCE

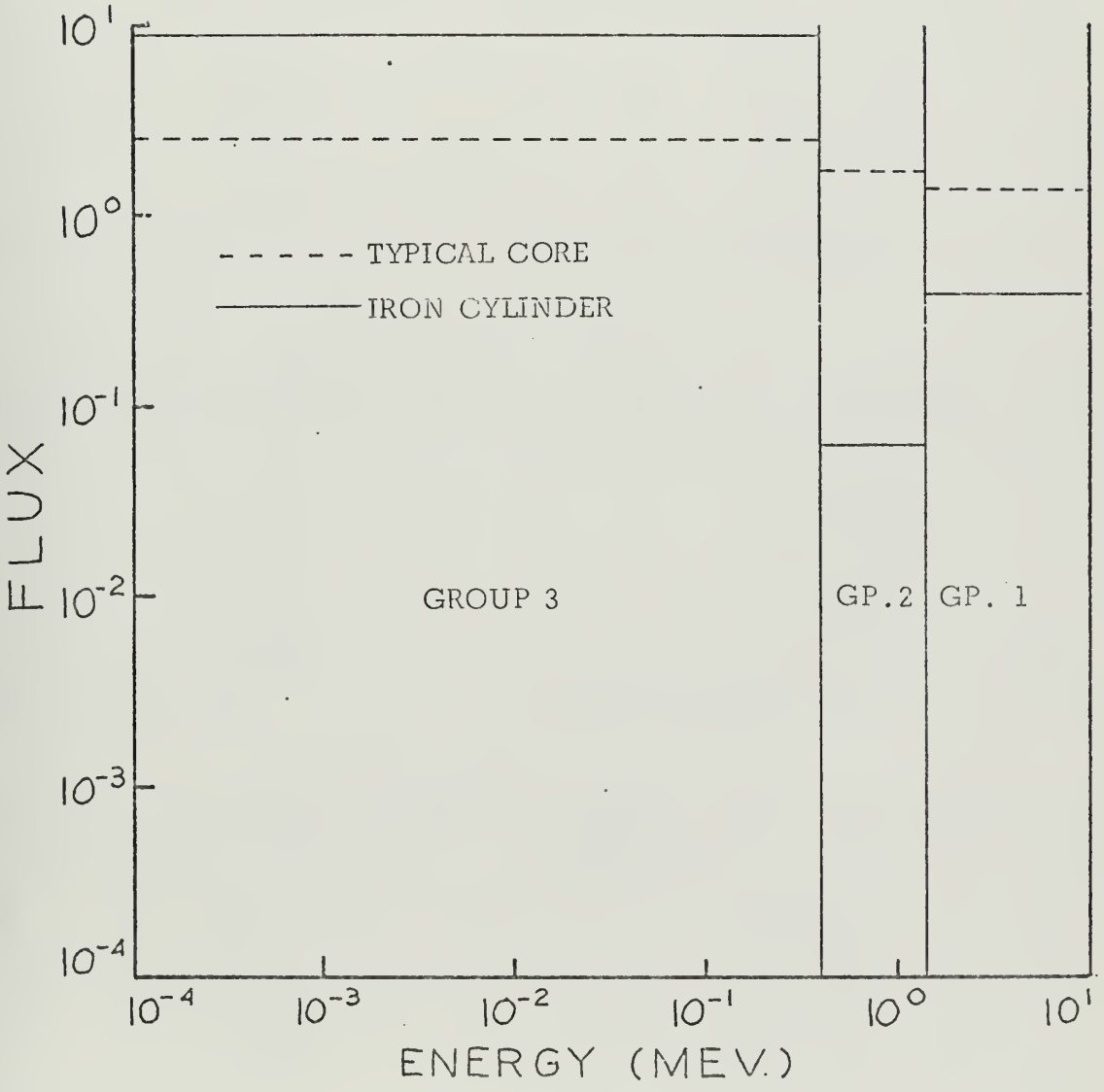


Figure IV-10

MESH SPACING = .75 CM.

VOL. PCT. U^{235}	K_{eff} AFTER 70 INNER ITERATIONS	OUTER ITERATION	EPSILON 1
2	.99464	0	1.8×10^{-4}
2.5	.99567	1	3.9×10^{-5}
3	.99668	2	3.7×10^{-5}
3.5	.99768	3	3.5×10^{-5}
4	.99865	4	3.3×10^{-5}
4.5	.99968	5	3.3×10^{-5}

CORE STORAGE 270K COMPUTER TIME 33:26

MESH SPACING = 1.5 CM.

VOL. PCT. U^{235}	K_{eff} AFTER 70 INNER ITERATIONS	OUTER ITERATION	EPSILON 1
2	.98952	0	4.7×10^{-8}
2.5	.99156	1	1.7×10^{-8}
3	.99355	2	2.5×10^{-8}
3.5	.99550	3	3.4×10^{-8}
4	.99741	4	4.2×10^{-8}
4.5	.99913	5	----------

CORE STORAGE 148K COMPUTER TIME 8:34

MESH SPACING = 2.0 CM.

VOL. PCT. U^{235}	K_{eff} AFTER 70 INNER ITERATIONS	OUTER ITERATION	EPSILON 1
2	.98631	0	1.1×10^{-9}
2.5	.98898	1	6.0×10^{-10}
3	.99159	2	6.0×10^{-10}
3.5	.99415	3	9.0×10^{-10}
4	.99665	4	1.2×10^{-9}
4.5	.99910	5	----------

CORE STORAGE 126K COMPUTER TIME 5:40

MESH SPACING = 4.0 CM.

VOL. PCT. U^{235}	K_{eff} AFTER 70 INNER ITERATIONS	OUTER ITERATION	EPSILON 1
2	.97542	0	10^{-10}
2.5	.98036	1	10^{-10}
3	.98517	2	10^{-10}
3.5	.98985	3	10^{-10}
4	.99442	4	10^{-10}
4.5	.99	5	----------

CORE STORAGE 100K COMPUTER TIME 1:41

Table IV-1

with the Bessel function and cosine respectively. Table IV-2 shows a maximum deviation of .71 per cent using a mesh size of two centimeters. Criticality was reached at the same core composition, i.e., 4.5% U^{235}, for mesh spacings of .75 centimeters, 1.5 centimeters and 2 centimeters. With a mesh size of four centimeters, criticality was not achieved at this composition.

The second method for arriving at a critical assembly consisted of fixing the core composition and varying the core dimensions in the outer iteration. A new optimum overrelaxation factor was computed for each outer iteration since the geometric parameters that were used to compute w_{opt} were changing. Three compositions corresponding to 18.9%, 20% and 27% enrichment were used. The results of this test are plotted in Figure IV-11 and compared with the bare core criticality curve of the RAPSODIE reactor in reference [11]. Differences noted are believed to be due to the following: a) compositions are not exactly the same; b) different cross section data was probably used though this was speculative since reference [11] did not state the source of their cross section data; and c) different programs were used to solve the multigroup diffusion equations. PRODII program was used for the RAPSODIE reactor. The factor above having the most influence was probably the large volume of iron. The absorption cross section of iron is fairly large and as seen in Table IV-3 is occupying a volume which in RAPSODIE contained 4% molybdenum, an unspecified quantity of iron in the form of stainless steel, and possibly some void space. The absorption cross section of these components is likely to be less than pure iron.

The comparison of these curves shows that while a much larger critical reactor, assuming the same enrichment, would result from using the program developed in this study, the shapes of the curves are very similar. The fact that as enrichment was increased, the core

MESH POINT LOCATION	INITIAL FLUX GROUPS 1, 2, 3	FINAL FLUX GROUPS 1,2, 3	PER CENT DEVIATION
(2,2)	1.000000	1.000000	--------
(4,2)	.982230	.981311	.090
(6,2)	.929858	.928161	.182
(8,2)	.845676	.843389	.270
(10,2)	.734131	.731515	.356
(12,2)	.601101	.598447	.441
(14,2)	.453505	.451134	.523
(16,2)	.298939	.297153	.597
(18,2)	.145212	.144270	.648
(20,2)	0.0	0.0	---------
(2,2)	1.000000	1.000000	--------
(2,4)	.984808	.983927	.089
(2,6)	.939696	.938015	.178
(2,8)	.866030	.863707	.268
(2,10)	.766040	.763312	.356
(2,12)	.642787	.639923	.445
(2,14)	.500000	.497327	.534
(2,16)	.342020	.339888	.623
(2,18)	.173646	.172411	.710
(2,20)	0.0	0.0	--------

$$\text{PER CENT DEVIATION} = \frac{\text{INITIAL FLUX} - \text{FINAL FLUX}}{\text{INITIAL FLUX}} \times 100$$

Table IV-2

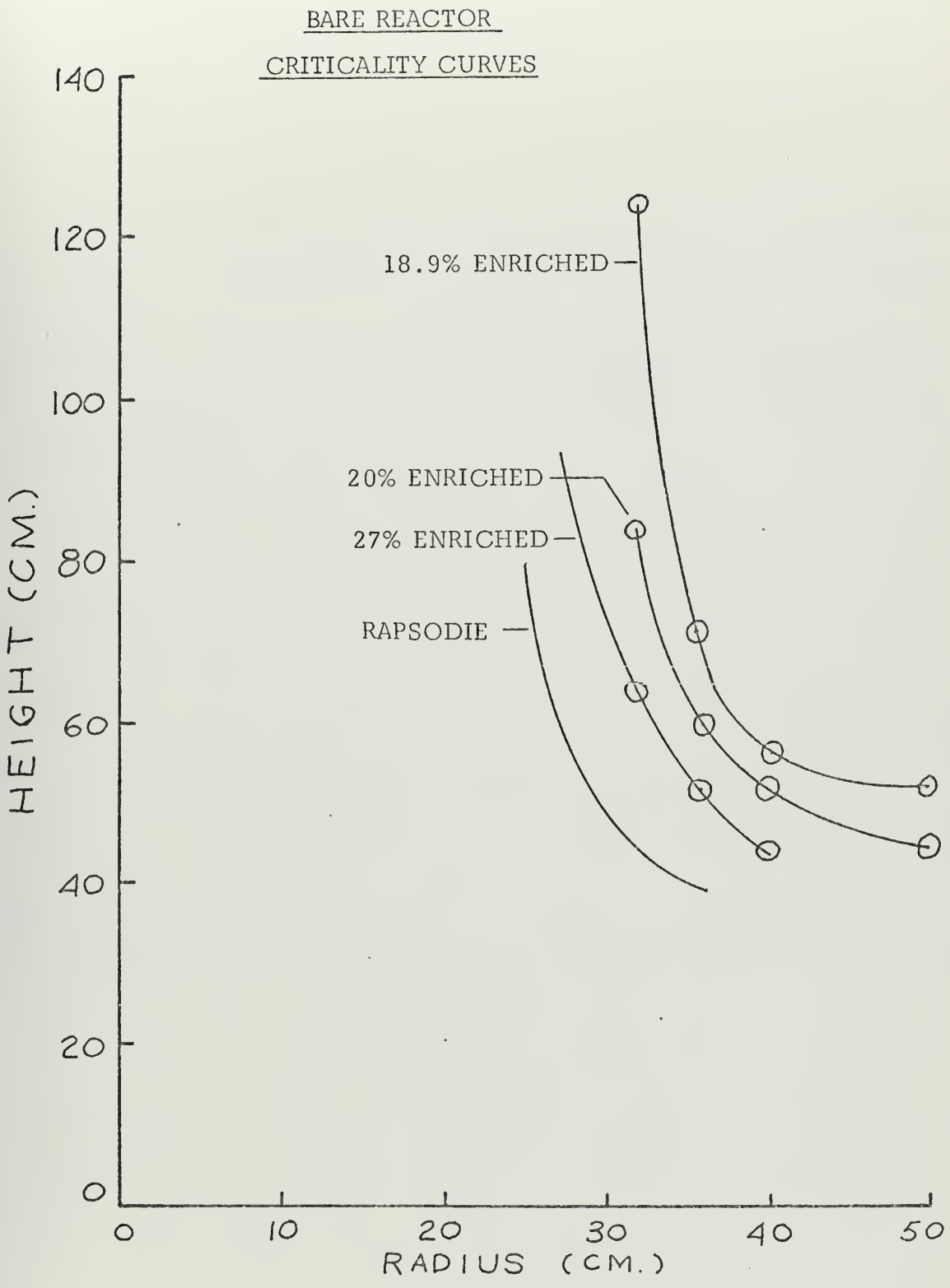

BARE REACTOR
CRITICALITY CURVES
18.9% ENRICHED
20% ENRICHED
27% ENRICHED
RAPSODIE
HEIGHT (CM.)
RADIUS (CM.)
0
20
40
60
80
100
120
140
0
10
20
30
40
50

Figure IV-11

	RAPSODIE	18.9% ENRICHED	20% ENRICHED
COMPUTER	IBM 650	IBM 360/67	IBM 360/67
COMPUTER CODE	PROD II	THIS STUDY	THIS STUDY
NUMBER OF GROUPS	?	3	3
CROSS SECTION DATA	?	ANL 3 GROUP	ANL 3 GROUP
COMPOSITION VOL. PERCENT			
URANIUM	28%	32%	32%
PLUTONIUM	8%	8%	8%
SODIUM	33%	33%	33%
MOLYBDENUM	4%	---	---
STAINLESS STEEL IRON	?	27%	27%

Table IV-3

size and hence the critical mass was decreased indicates a correct trend. The flux shapes found using this outer iteration scheme were also nearly coincident with the cosine and Bessel function curves. Table IV-4 shows an example of the per cent deviations experienced for a critical core assembly 32 centimeters in radius and 124 centimeters in height.

E. TESTS WITH SIXTEEN GROUP DATA

Attempts to modify the program to solve a sixteen group problem were not successful. Both under- and over-relaxation factors were tried but satisfactory convergence of the inner iteration could not be achieved after over thirty minutes of computer time. An under-relaxation factor of .1 was used and after 310 iterations and 34 minutes, the convergence criterion was only of the order 10^{-1}. Overrelaxing with a factor of 1.726 yielded a highly oscillatory flux behavior in all energy groups. All group fluxes were initially estimated to have maximum value of ten. The magnitude of oscillations recorded in the output varied between plus or minus eighty for the higher energy groups to plus or minus thirty for the lower energy groups with no noticeable damping after 310 iterations and some 34 minutes of computer time. No further tests were conducted.

MESH POINT LOCATION	INITIAL FLUX GROUPS 1, 2, 3	FINAL FLUX GROUPS 1, 2, 3	PER CENT DEVIATION
(2,2)	1.000000	1.000000	---
(4,2)	.977535	.977479	.006
(6,2)	.911646	.911554	.010
(8,2)	.806757	.806654	.013
(10,2)	.669888	.669786	.015
(12,2)	.510080	.510013	.013
(14,2)	.337808	.337782	.008
(16,2)	.164121	.164154	.020
(18,2)	0.0	0.0	---
(2,2)	1.000000	1.000000	---
(2,4)	.994868	.994805	.006
(2,6)	.979525	.979403	.012
(2,8)	.954141	.953952	.020
(2,10)	.918959	.918717	.026
(2,12)	.874343	.874059	.032
(2,14)	.820767	.820438	.040
(2,16)	.758757	.758407	.046
(2,18)	.688969	.688603	.053
(2,20)	.612101	.611742	.058
(2,22)	.528959	.528615	.065
(2,24)	.440393	.440076	.072
(2,26)	.347303	.347033	.078
(2,28)	.250656	.250440	.086
(2,30)	.151424	.151289	.089
(2,32)	.050646	.050600	.091

Table IV-4

V. CONCLUSIONS

This study was undertaken to develop an easy to apply computer code for numerically solving the multigroup diffusion equations. The program was written in the FORTRAN IV language and all work was done on an IBM 360/67 computer. Applicability to other computer systems, while not a consideration of this study, is assumed to be practical.

The unique feature of this program is that it will be readily available to and easily used by students at the Naval Postgraduate School. It is hoped that this study will complement future local experimental studies in fast neutron diffusion and also enable the student researchers to conserve valuable time which would be expended attempting to verify their experimental data with a myriad of complicated and unfamiliar multigroup diffusion programs.

The point successive overrelaxation method of solution was successful with the three group data on the basis of the convergence criterion chosen, i.e.,

$$\left| \phi^{N-1}(E,r,z) - \phi^{N}(E,r,z) \right|_{\substack{ALL \\ E,r,z}} < 10^{-6}$$

Convergence of the three group criticality problem with five outer iterations took five minutes and forty seconds and 126K words of core storage using a mesh size of two centimeters. Criticality was achieved with smaller mesh sizes, i.e., .75 centimeters and 1.5 centimeters, but computation times were considered excessive especially for .75 centimeter mesh size which took about 33 minutes of computer time and over twice as much core storage space.

Tests showed that the code was easily adaptable to three group solutions for typical fast reactor cores and reactor material assemblies with extraneous point sources. Flux shapes and energy spectrums from

these tests should provide a sound basis for future experimental flux measuring studies in fast reactor materials.

The close coincidence of the critical flux shapes with the cosine and Bessel functions indicates that for a three group problem if critical core size and composition are known, then the space-independent solution using the buckling may be used. It was found, however, that this was not true for subcritical assemblies, i.e., the space and energy dependence of the flux are not separable.

Further work with larger numbers of energy groups will have to be carried out to establish the program's ability to handle successfully many group calculations. The unsuccessful attempt at using sixteen groups was due to the failure to attain suitable convergence in a reasonable amount of time. A better technique than the point successive over-relaxation method is obviously needed when dealing with this number of energy groups in order to improve the convergence time.

VI. RECOMMENDATIONS FOR FUTURE WORK

In its present form the program has proven to be successful for the purpose for which it was intended. Future researchers might choose to expand or modify this program in one or more of the following areas.

A. GEOMETRY AND SPATIAL DIMENSIONS

Although the program is written for the cylindrical geometry in two spatial dimensions (flux ϕ is not a function of azimuth angle θ), it need not be so limited. The program could easily be adapted to the three dimensional cylindrical geometry as well as the two or three dimensional rectangular geometry. The addition of a third dimension in space would, of course, require considerably more computer core storage and hence the future user might have to limit the physical size and number of energy groups in a particular problem.

Another possible additional feature might be to increase the number of regions by one or more to include blanket, reflector and shielding regions.

B. CROSS SECTION DATA

All the results for this program were obtained using three group ANL data. Conceivably future users would want to use the most current and widely accepted cross section available to them. For this reason the program could be modified to handle multigroup data of up to twenty-six groups or higher. Once again the user must bear in mind that as the number of energy groups is increased, core storage and computation time will also increase and probably not linearly.

C. INNER ITERATION METHODS

The point successive overrelaxation iterative method gave satisfactory convergence rates for the three group problems used in

testing the program. However, more rapid convergence of the inner iteration is possible [15] using line rather than point successive overrelaxation methods and cyclic Chebyshev semi-iterative methods. If the future user desires to use more than three energy groups, he will almost certainly have to change the program to fit one of these methods of solution.

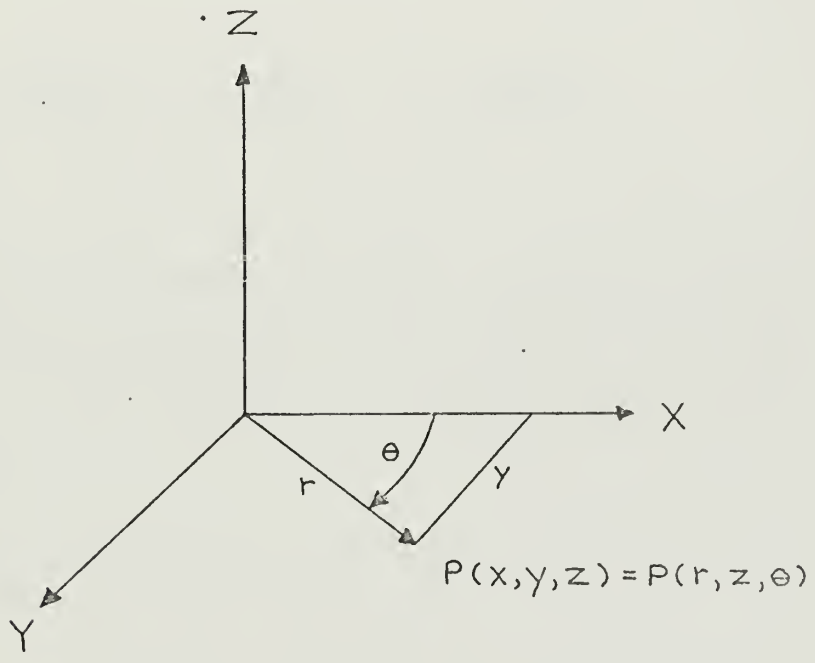
D. TIME DEPENDENCE

On a somewhat more far reaching level, this program might be expanded to include the time domain for the purpose of studying the dynamics of a fast reactor. This could also include a study of space-dependent burnup and reactivity feedback effects. These topics are currently under investigation by those working on the development of the fast breeder reactor.

APPENDIX A

DERIVATION OF LAPLACIAN OPERATOR IN CYLINDRICAL COORDINATES AND FINITE DIFFERENCE APPROXIMATIONS

In order to numerically solve the set of partial differential equations II-31, a finite difference technique was chosen. In cylindrical geometry the Laplacian or ∇^2 operator is approximated by transforming the Cartesian coordinates to polar coordinates.



$$x = r\cos\theta \quad ; \quad y = r\sin\theta$$

$$r = (x^2 + y^2)^{1/2} \quad ; \quad \theta = \tan^{-1}\frac{y}{x} \quad . \qquad \text{A-1}$$

Taking the partial derivations of r and θ with respect to x and y in Equation A-1 results in

$$r_x = \frac{1}{2} \quad \frac{2x}{(x^2+y^2)^{1/2}} = \frac{x}{r} = \cos\theta \quad , \qquad \text{A-2}$$

$$r_y = \frac{1}{2} \quad \frac{2y}{(x^2+y^2)^{1/2}} = \frac{y}{r} = \sin\theta \quad , \qquad \text{A-3}$$

$$\theta_x = \frac{y/x^2}{1+(y/x)^2} = -\frac{y}{r^2} = -\frac{\sin\theta}{r} \quad , \qquad \text{A-4}$$

$$\theta_y = \frac{1/x}{1+(y/x)^2} = \frac{x}{r^2} = \frac{\cos\theta}{r} \quad . \qquad \text{A-5}$$

Now consider a function $\phi(r,z,\theta)$, wherein r and θ are functions of x and y through Equation A-1. Taking the first partial derivative of ϕ with respect to x, y, and z utilizing Equations A-2 through A-5 gives

$$\phi_x = \phi_r r_x + \phi_\theta \theta_x = \phi_r \cos\theta - \phi_\theta \frac{\sin\theta}{r} \quad , \qquad \text{A-6}$$

$$\phi_y = \phi_r r_y + \phi_\theta \theta_y = \phi_r \sin\theta + \phi_\theta \frac{\cos\theta}{r} \quad , \qquad \text{A-7}$$

$$\phi_z = \phi_z Z_z = \phi_z \quad . \qquad \text{A-8}$$

The second partial derivations are now obtained as follows through Equations A-6 through A-8:

$$\phi_{xx} = \left(\frac{\partial}{\partial r}\cos\theta - \frac{\partial}{\partial\theta}\frac{\sin\theta}{r}\right)\left(\phi_r\cos\theta - \phi_\theta\frac{\sin\theta}{r}\right)$$

$$= \phi_{rr}\cos^2\theta + \phi_r\frac{\sin^2\theta}{r} + \phi_{\theta\theta}\frac{\sin^2\theta}{r^2} - 2\phi_{r\theta}\frac{\sin\theta\cos\theta}{r} + 2\phi_\theta\frac{\sin\theta\cos\theta}{r^2}, \qquad \text{A-9}$$

$$\phi_{yy} = \left(\frac{\partial}{\partial r}\sin\theta + \frac{\partial}{\partial\theta}\frac{\cos\theta}{r}\right)\left(\phi_r\sin\theta + \phi_\theta\frac{\cos\theta}{r}\right)$$

$$= \phi_{rr}\sin^2\theta + \phi_r\frac{\cos^2\theta}{r} + \phi_{\theta\theta}\frac{\cos^2\theta}{r} + 2\phi_{r\theta}\frac{\cos\theta\sin\theta}{r} - 2\phi_\theta\frac{\cos\theta\sin\theta}{r^2}, \qquad \text{A-10}$$

$$\phi_{zz} = \frac{\partial}{\partial z}(\phi_z) = \phi_{zz}. \qquad \text{A-11}$$

Adding Equations A-9, A-10 and A-11, the Laplacian in cylindrical coordinates becomes

$$\nabla^2\phi = \phi_{rr} + \frac{1}{r}\phi_r + \frac{1}{r^2}\phi_{\theta\theta} + \phi_{zz} \qquad \text{A-12}$$

The partial derivatives are replaced by their respective central difference operators [16] with errors of order h^2 where h^2 is the square of the distance between adjacent mesh points (See Figure A-1.) resulting in

$$\nabla^2\phi = \frac{1}{h^2}\{\phi(r+1,z,\theta) - 2\phi(r,z,\theta) + \phi(r-1,z,\theta)\}$$

$$+ \frac{1}{2rh}\{\phi(r+1,z,\theta) - \phi(r-1,z,\theta)\} + \frac{1}{r^2\gamma^2}\{\phi(r,z,\theta-1)$$

$$- 2\phi(r,z,\theta) + \phi(r,z,\theta+1)\} + \frac{1}{h^2}\{\phi(r,z+1,\theta) - 2\phi(r,z,\theta)$$

$$+ \phi(r,z-1,\theta)\}. \qquad \text{A-13}$$

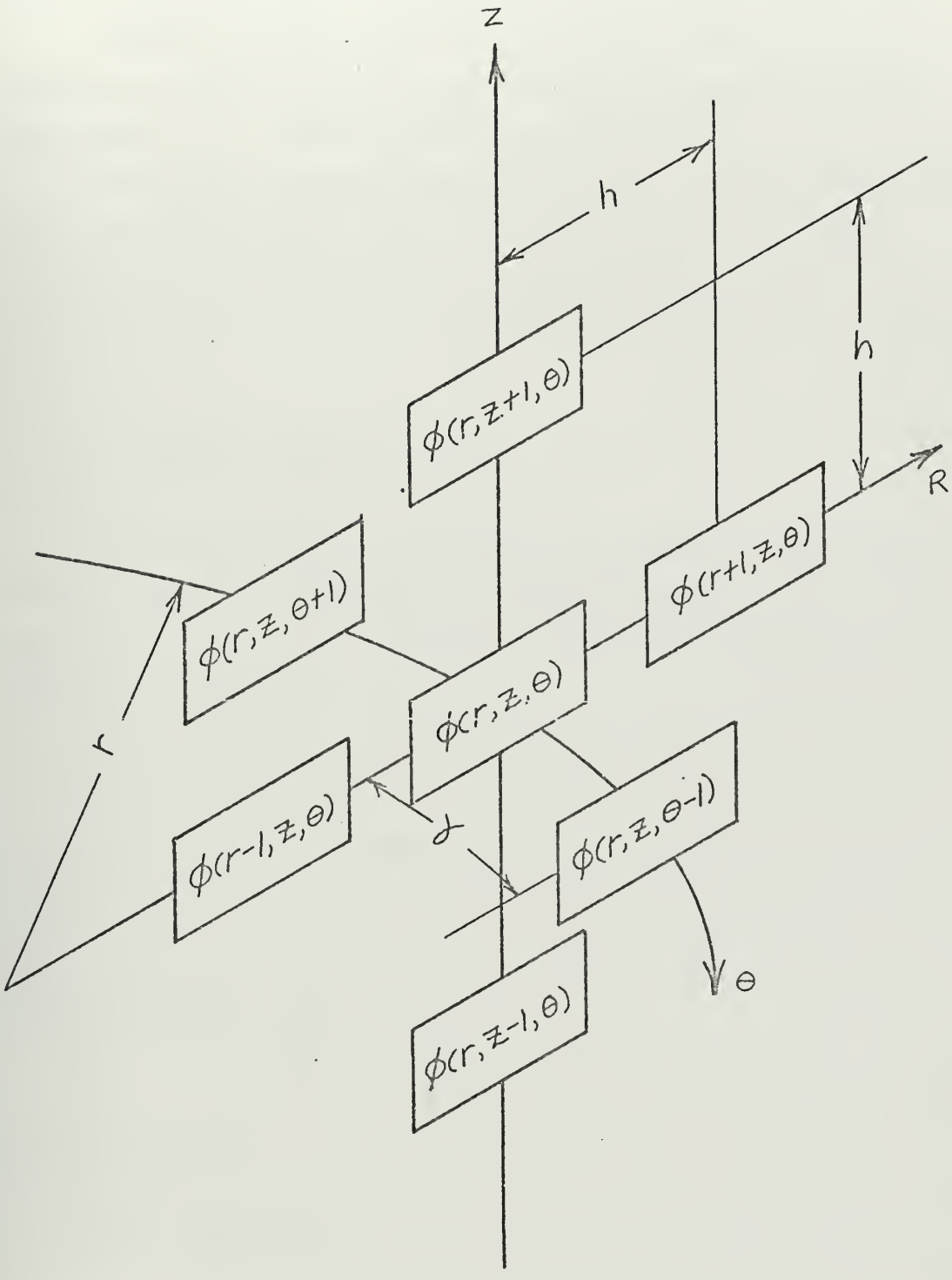


Figure A-1

where the symbols used in Equation A-13 are shown in Figure A-1.

Salvadori and Baron [16], as well as any other good text on numerical methods, derives finite difference operators with higher order errors, i.e., more accurate approximations.

It was found that this operator cannot be used for a mesh point lying on the longitudinal axis of the cylinder. At these points Equation A-12 has singularities in the second and third terms. In this study circular symmetry was assumed, thus Equation A-12 reduced to

$$\nabla^2 \phi = \phi_{rr} + \frac{1}{r}\phi_r + \phi_{zz} \quad . \qquad \text{A-14}$$

There still existed a singularity in the second term; however, the problem under investigation also possessed symmetry with respect to the origin, i.e., $\phi_r = 0$ at $r = 0$. Smith [9] shows that by Maclaurin's expansion

$$\phi_r(r) = \phi_r(0) + r\,\phi_{rr}(0) + \frac{1}{2} r^2 \phi_{rrr}(0) + \cdots \quad , \qquad \text{A-15}$$

however, $\phi_r(0) = 0$. Therefore, as r approaches zero in the limit the term $\frac{1}{r}\phi_r$ in Equation A-14 is the second partial derivative of ϕ at $r = 0$ plus higher order terms. Equation A-14 can be written for mesh points on the longitudinal axis

$$\nabla^2 \phi = 2\phi_{rr} + \phi_{zz} \qquad \text{A-16}$$

or in finite difference operator form

$$\nabla^2 \phi = \frac{2}{h^2}\left\{\phi(r+1,z) - 2\phi(r,z) + \phi(r-1,z)\right\} + \frac{1}{h^2}\left\{\phi(r,z+1) - 2\phi(r,z) + \phi(r,z-1)\right\} . \qquad \text{A-17}$$

APPENDIX B

DEFINITION OF COEFFICIENTS

In order to simplify the writing of Equations II-35, they are first expanded in order to utilize the sixteen group data available in reference 7 .

A typical equation, for example, group three, is written as follows:

$$\underbrace{\left[\sum_{J=1}^{C} N^J \left(\sigma^J_{in,1\rightarrow 3} + \chi_3 \nu_1^J \sigma^J_{f,1}\right)\right]}_{A_3(1)} \phi_1(r,z)$$

$$+ \underbrace{\left[\sum_{J=1}^{C} N^J \left(\sigma^J_{in,2\rightarrow 3} + \chi_3 \nu_2^J \sigma^J_{f,2}\right)\right]}_{A_3(2)} \phi_2(r,z)$$

$$+ \underbrace{\frac{1}{3\sum_{J=1}^{C} N^J \sigma^J_{tr,3}}}_{A_3(3)} \left[\frac{1}{h^2}\left\{\phi_3(r+1,z) - 2\phi_3(r,z) + \phi_3(r-1,z)\right\}\right.$$

$$+ \frac{1}{2rh}\left\{\phi_3(r+1,z) - \phi_3(r-1,z)\right\} + \frac{1}{h^2}\left\{\phi_3(r,z+1) - 2\phi_3(r,z)\right.$$

$$\left.\left. + \phi_3(r,z-1)\right\}\right] - \underbrace{\left[\sum_{J=1}^{C} N^J \left(\sigma^J_{c,3} + \sigma^J_{in,3\rightarrow 3} + \sigma^J_{in,3\rightarrow 4}\right.\right.}$$

$$\underbrace{+ \sigma^J_{in,3\rightarrow 5} + \sigma^J_{in,3\rightarrow 6} + \sigma^J_{in,3\rightarrow 7} + \sigma^J_{in,3\rightarrow 8}}_{A_3(4)}$$

$$\left.\left. + \sigma^J_{er,3} + \sigma^J_{f,3}\right)\right] \phi_3(r,z)$$

$$+\underbrace{\left[\sum_{J=1}^{C} N^J (\chi_3 \nu_3^J \sigma_{f,3}^J)\right]}_{A_3(5)} \phi_3(r,z) + \cdots$$

$$+\underbrace{\left[\sum_{J=1}^{C} N^J (\chi_3 \nu_{16}^J \sigma_{f,16}^J)\right]}_{A_3(18)} \phi_{16}(r,z) = S_3(r,z) \qquad \text{B-1}$$

This equation is then written in the form

$$\left[A_3(4) + A_3(3)\left(\frac{4}{h^2}\right)\right]\phi_3(r,z) + \left[A_3(3)\left(\frac{1}{h^2} + \frac{1}{2rh}\right)\right]$$

$$\phi_3(r+1,z) + \left[A_3(3)\left(\frac{1}{h^2}\right)\right]\phi_3(r,z-1) + \cdots = A_3(1)\phi_1(r,z)$$

B-2

$$+ A_3(2)\,\phi_2(r,z) + A_3(5)\,\phi_3(r,z) + \cdots$$

$$+ A_3(18)\,\phi_{16}(r,z) + S_3(r,z)$$

The general form of this equation is

$$\left[A_g(g+1) + A_g(g)\left(\frac{4}{h^2}\right)\right]\phi_g(r,z) + \left[A_g(g)\left(\frac{1}{h^2} + \frac{1}{2rh}\right)\right]$$

$$\phi_g(r+1,z) + \left[A_g(g)\left(\frac{1}{h^2} - \frac{1}{2rh}\right)\right]\phi_g(r-1,z) + \left[A_g(g)\right.$$

$$\left.\left(\frac{1}{h^2}\right)\right]\phi_g(r,z+1) + \left[A_g(g)\left(\frac{1}{h^2}\right)\right]\phi_g(r,z-1) \qquad \text{B-3}$$

$$= A_g(1)\,\phi_1(r,z) + \cdots + A_g(g-1)\,\phi_{g-1}(r,z)$$

$$+ A_g(g+2)\,\phi_g(r,z) + \cdots + A_g(18)\,\phi_{16}(r,z)$$

$$+ S_g(r,z)\,.$$

This general equation is condensed further to the form

$$A1\phi_g(r-1,z)+A2\,\phi_g(r,z)+A3\,\phi_g(r+1,z)+A4\,\phi_g(r,z-1)$$
$$+A5\,\phi_g(r,z+1)=AS_1\,\phi_1(r,z)+\cdots+AS_{g-1}\,\phi_{g-1}(r,z) \qquad \text{B-4}$$
$$+AF_g\,\phi_g(r,z)+\cdots+AF_{18}\,\phi_{16}(r,z)+S_g(r,z).$$

In energy group one, Equation B-3 does not apply. The reason for this is that the terms on the right side of Equation B-3 are not fully applicable to group one. These terms have the following meaning: the sources of all neutrons that enter group g due to down scattering from all higher energy groups plus neutrons born into group g due to fissions in all higher energy groups are represented by the first set of terms on the right hand side of Equation B-3 $[Ag(1)\,\phi_1(r,z)+\cdots+Ag(g-1)\,\phi_{g-1}(r,z)]$. In group one these terms must be zero since there are no higher energy groups.

These apparent inconsistencies were accounted for in the computer program, i.e., these terms were set to zero in the first group equation.

APPENDIX C

FINITE DIFFERENCE MESH POINT DESIGNATION SYSTEM

In order to take advantage of symmetry in the study and hence reduce the computer core storage space needed, the following mesh point scheme was adopted: a) all points $\phi(1,x) = \phi(3,x)$ and $\phi(x,1) = \phi(x,3)$ due to symmetry, b) all extrapolated boundary points $\phi(L,x) = \phi(M,x) = 0$ and c) no mesh points have zero index due to computer language incompatability. See Figure C-1 for further details.

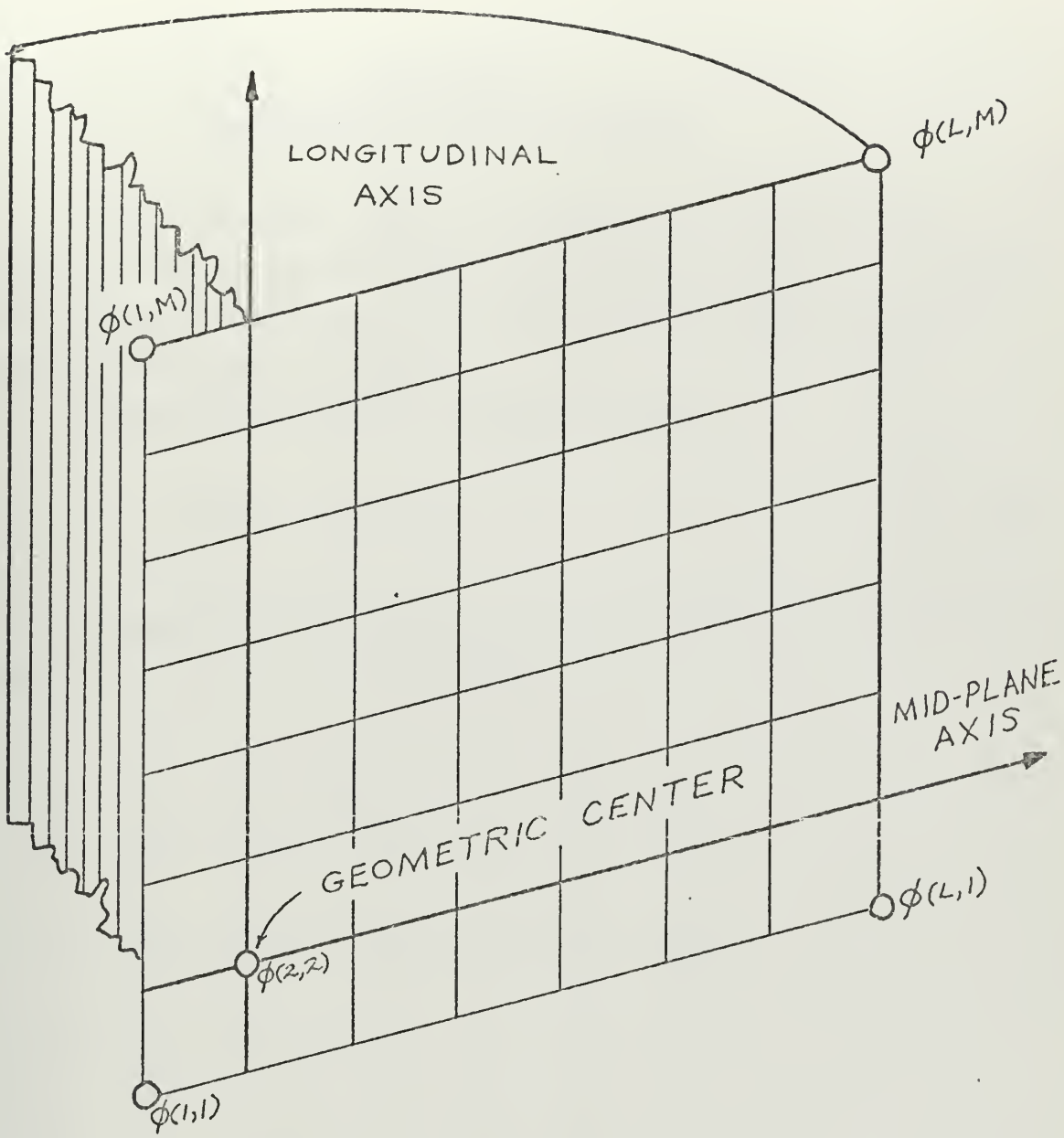

LONGITUDINAL
AXIS
$\phi(L,M)$
$\phi(1,M)$
MID-PLANE
AXIS
GEOMETRIC CENTER
$\phi(L,1)$
$\phi(2,2)$
$\phi(1,1)$

Figure C-1

APPENDIX D

GAUSS-SEIDEL ITERATIVE METHOD

In this technique, which is also called the unextrapolated Liebmann method, the point single step iterative method, or the method of successive displacements, the latest iterative values are used as soon as they become available.

Consider the solution of Poisson's equation

$$\nabla^2 \phi = f(x,y) \,, \qquad \text{D-1}$$

in a rectangular region as in Figure D-1. Assume the solution is proceeding column by column from left to right.

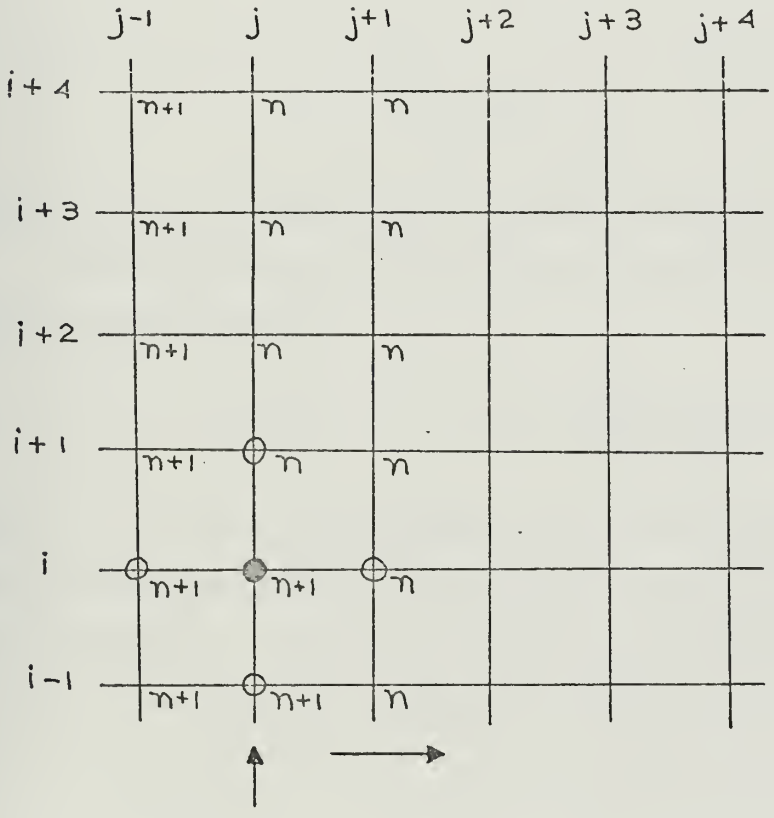


Figure D-1

If the point at which the equation is presently being solved is $\phi(i,j)$, the iteration formula is

$$\phi^{n+1}(i,j) = \frac{1}{4}\{ \phi^{n+1}(i-1,j) + \phi^{n}(i+1,j) + \phi^{n+1}(i,j-1) + \phi^{n}(i,j+1) + h^2 f(i,j) \} , \qquad \text{D-2}$$

where the standard five-point central difference approximation is substituted for the ∇^2 operator.

POINT SUCCESSIVE OVERRELAXATION

This method, also called the extrapolated Leibmann method, can be thought of as the sum of the Gauss-Seidel iterate Equation D-2 and the nth iterate. It is written as

$$\phi^{n+1}(i,j) = w\left[\frac{1}{4}\{ \phi^{n+1}(i-1,j) + \phi^{n}(i+1,j) + \phi^{n+1}(i,j-1) + \phi^{n}(i,j+1) + h^2 f(i,j) \}\right] + (1-w)\,\phi^{n}(i,j) , \qquad \text{D-3}$$

where w is the overrelaxation or acceleration factor.

The optimum value of this factor in general is $1 < w_{opt} < 2$ and can be computed from

$$W_{opt} = \frac{2}{1+\sqrt{1-\lambda_1^2}} . \qquad \text{D-4}$$

where λ_1 is the largest eigenvalue of the problem resulting from a solution using the method of successive displacements, i.e., the Jacobian method. In practice, however, the eigenvalues are seldom known and finding them by solving the problem by successive displacements can be very difficult due to slow convergence. References 8, 9, 10, 17 recommend an experimental process for determining w_{opt} for a particular problem. In this process various values of w are tried until one is found that gives the most rapid convergence. This is then determined

to be w_{opt}. Reference 16 suggests two other methods for determining w_{opt}.

The experimental approach described above was used in this study.

The rapid convergence experienced with this method on many different problems has proved far superior to the ordinary Gauss-Seidel or successive displacements method without overrelaxation. References 8, 17 contain some examples.

References 8, 9, 10, 17 relate more explicit details on the method of successive overrelaxation in varying degrees.

APPENDIX E

INPUT DATA SYMBOLS

SYMBOL	NAME & DESCRIPTION	UNITS	REMARKS
GS	MESH SPACING	CM.	VALUES FROM .75 TO 2.0 HAVE BEEN USED SUCCESS-FULLY
GSSQ	VALUE OF MESH SPACING SQUARED	$CM.^2$	
C	TOTAL NUMBER OF ELEMENTS OR ISOTOPES IN THE ASSEMBLY		UP TO FIVE HAVE BEEN USED
R	NUMBER OF MESH POINTS IN THE RADIAL DIRECTION PLUS ONE		ADDITIONAL POINT LIES OUTSIDE REGION OF SOLU-TION BUT IS NEEDED FOR DERIV-ATIVE BOUNDARY CONDITION
Z	NUMBER OF MESH POINTS IN THE AXIAL DIRECTION PLUS ONE		IS IN EFFECT ONLY HALF THE AXIAL DIMENSION
SS(I)	ENERGY DEPENDENT SOURCE	$NEU./CM.^2$	SPACE DEPENDENCE MUST BE ACCOUNT-ED FOR IN DIFFUSION EQUATIONS
S(I)	GROUP RELATIVE MAGNITUDE FACTOR		
DELE(I)	ENERGY GROUP WIDTH	Mev	
H(I)	ATOMIC ABUNDENCE OF EACH ELEMENT OR ISOTOPE	$ATOM/CM.^3$	
SIGIN (I,J,K)	INELASTIC SCATTER-ING CROSS SECTION OF THE Ith ELEMENT FROM GP.J TO GP.K	barns	

SYMBOL	NAME & DESCRIPTION	UNITS	REMARKS
SIGTR(I,J)	THE TRANSPORT CROSS SECTION OF THE Ith ELEMENT OR ISOTOPE AT THE Jth ENERGY LEVEL	barns	
SIGF(I,J)	THE FISSION CROSS SECTION OF THE Ith FISSIONABLE ELEMENT OR ISOTOPE AT THE Jth ENERGY LEVEL	barns	
SIGC(I,J)	THE CAPTURE CROSS SECTION OF THE Ith ELEMENT OR ISOTOPE AT THE Jth ENERGY LEVEL	barns	
CHI(I)	NORMALIZED FISSION SOURCE FOR THE Ith ENERGY LEVEL		
NU(I,J)	AVERAGE NUMBER OF NEUTRONS RELEASED PER FISSION OF THE Ith FISSIONABLE ELEMENT OR ISOTOPE AT THE Jth ENERGY LEVEL		

INPUT DATA

DELE(I)

8.65000 0.95000 0.40000

H(I)

0.000980 0.014500 0.008460 0.022800 0.003960

SIGIN(I,J,K)

0.0	1.0000	0.5000	0.0	0.0
0.0	0.0	0.5000	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	1.4100	0.6400	0.0	0.0
0.0	0.0	0.2500	0.0	0.0
0.0	0.0	0.1	0.0	0.0
0.0	0.2400	0.0600	0.0	0.0
0.0	0.0	0.1800	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.6000	0.1000	0.0	0.0
0.0	0.0	0.2200	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.4500	0.4500	0.0	0.0
0.0	0.0	0.5000	0.0	0.0
0.0	0.0	0.0	0.0	0.0

SIGTR(I,J)

4.50000 5.70000 10.00000
4.60000 5.80000 9.60000
2.00000 3.20000 3.70000
2.30000 2.13000 3.24000
4.60000 6.00000 10.00000

SIGF(I,J)

1.29000 1.27000 1.77000
0.52400 0.01000 0.0
1.95000 1.75000 1.80000

SIGC(I,J)

0.08000 0.13000 0.49000
0.03600 0.13000 0.26000
0.00050 0.00100 0.00100
0.00500 0.00600 0.00600
0.10000 0.12000 0.35000

CHI(I)

0.57500 0.32600 0.09900

NU(I,J)

2.70000	2.53000	2.47000
2.60000	2.47000	0.0
3.10000	2.98000	2.91000

A(I,J)

2.195207	-0.070956	0.027086	0.013891	0.014390
0.054274	1.762904	-0.022586	0.007876	0.008159
0.019003	0.015025	1.134659	-0.014644	0.002478

INITIAL FLUX ESTIMATE

PHI(I,R,2)

0.99000	1.60000	2.33000
0.97241	1.57156	2.28859
0.92056	1.48777	2.16657
0.83722	1.35308	1.97042
0.72679	1.17461	1.71052
0.59509	0.96175	1.40055
0.44897	0.72560	1.05666
0.29595	0.47830	0.69653
0.14376	0.23235	0.33835
0.0	0.0	0.0

PHI(I,2,Z)

0.99000	1.60000	2.33000
0.97496	1.57569	2.29460
0.93030	1.50351	2.18948
0.85737	1.38564	2.01784
0.75838	1.22567	1.78488
0.63636	1.02846	1.49770
0.49500	0.80000	1.16500
0.33860	0.54723	0.79691
0.17191	0.27784	0.40460
0.0	0.0	0.0

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.011322380084	0.988804380970	0

PHI(I,R,2)

1.03307536	1.60425648	2.41554146
1.00668525	1.55887732	2.35287141
0.94636000	1.46225915	2.21109566
0.85493754	1.31845326	1.99679573
0.73706743	1.13450394	1.72083461
0.59875409	0.91962149	1.39726136
0.44723283	0.68570746	1.04337095
0.29140791	0.44651821	0.67971301
0.12979276	0.21418796	0.32607146
0.0	0.0	0.0

PHI(I,2,Z)

1.03307536	1.60425648	2.41554146
1.01006291	1.56439369	2.36081468
0.95787500	1.48062882	2.23809102
0.87793766	1.35477253	2.05064529
0.77242182	1.18996665	1.80354443
0.64426762	0.99061647	1.50365684
0.49726229	0.76288212	1.15997627
0.33666590	0.51579907	0.78512947
0.16882065	0.25850988	0.39366809
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0067402814	1.726	2.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
20	1.013381515881	0.986795184566	0

PHI(I,R,2)

1.05760131	1.62174418	2.46817466
1.02561180	1.57235672	2.39324385
0.95884508	1.46987651	2.23720961
0.86103764	1.31995461	2.00877660
0.73783093	1.13116625	1.72128072
0.59623133	0.91411096	1.39086791
0.44387702	0.68055229	1.03542053
0.28869424	0.44264733	0.67341506
0.13838928	0.21221396	0.32281437
0.0	0.0	0.0

PHI(I,2,Z)

1.05760131	1.62174418	2.46817466
1.02922308	1.57788363	2.40167212
0.97060308	1.48771257	2.26455762
0.88386111	1.35462497	2.06189036
0.77217407	1.18344436	1.80112861
0.63959976	0.98032178	1.49175804
0.49086706	0.75245619	1.14480590
0.33113988	0.50773454	0.77229593
0.16577749	0.25434055	0.38669710
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0027816363	1.726	2.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.013688017085	0.986496814745	0

PHI(I,R,2)

1.06234136	1.62906304	2.47809042
1.02869485	1.57746549	2.39958579
0.96054918	1.47296515	2.24062202
0.86173130	1.32143591	2.01012798
0.73796143	1.13159045	1.72140446
0.59607804	0.91401889	1.39044061
0.44366317	0.68030605	1.03490999
0.28853712	0.44243755	0.67305499
0.13831788	0.21209375	0.32264637
0.0	0.0	0.0

PHI(I,2,Z)

1.06234136	1.62906304	2.47809042
1.03213599	1.58274314	2.40760887
0.97162246	1.48991638	2.26639917
0.88340690	1.35468160	2.06069492
0.77091797	1.18207620	1.79825747
0.63814688	0.97850510	1.48855583
0.48963374	0.75086127	1.14224660
0.33243397	0.50667515	0.77077712
0.16549597	0.25376580	0.38603948
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0001426653	1.726	2.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
40	1.013701871159	0.986483332478	0

PHI(I,R,2)

1.06253205	1.62925519	2.47850564
1.02882635	1.57757321	2.39988061
0.96064980	1.47303376	2.24084836
0.86180123	1.32146228	2.01026989
0.73799485	1.13162101	1.72147422
0.59609474	0.91403531	1.39047258
0.44366957	0.68031082	1.03491994
0.28853784	0.44243612	0.67305388
0.13831679	0.21209134	0.32264277
0.0	0.0	0.0

PHI(I,2,Z)

1.06253205	1.62925519	2.47850564
1.03225028	1.58282318	2.40786715
0.97165307	1.48990522	2.26651422
0.88338609	1.35454980	2.06060443
0.77083587	1.18197757	1.79807918
0.63806620	0.97839226	1.48837570
0.48961864	0.75076702	1.14210169
0.33039162	0.50661286	0.77068312
0.16547657	0.25373722	0.38599645
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000090142	1.726	2.0%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
50	1.013702430799	0.986482787865	0

PHI(I,R,2)

1.06254386	1.62927572	2.47852863
1.02883473	1.57758703	2.39989743
0.96065541	1.47304272	2.24085984
0.86180469	1.32146773	2.01027703
0.73799679	1.13162405	1.72147820
0.59609569	0.91403676	1.39047452
0.44366992	0.68031127	1.03492062
0.28853787	0.44243605	0.67305394
0.13831676	0.21209112	0.32264271
0.0	0.0	0.0

PHI(I,2,Z)

1.06254386	1.62927572	2.47852863
1.03225728	1.58283505	2.40788099
0.97165470	1.48990858	2.26651713
0.88337763	1.35454702	2.06059875
0.77083124	1.18197151	1.79806894
0.63806128	0.97838563	1.48836490
0.48961466	0.75076164	1.14209292
0.33038909	0.50660939	0.77067737
0.16547546	0.25373551	0.38599406
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000005421	1.726	2.0%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
60	1.012702480595	0.986482739406	0

PHI(I,R,2)

1.06254469	1.62927681	2.47853039
1.02883531	1.57758777	2.39989865
0.96065579	1.47304318	2.24086063
0.86180492	1.32146797	2.01027750
0.73799691	1.13162416	1.72147841
0.59609573	0.91403681	1.39047458
0.44366993	0.68031127	1.03492060
0.28853786	0.44243603	0.67305391
0.13831675	0.21209110	0.32264268
0.0	0.0	0.0

PHI(I,2,Z)

1.06254469	1.62927681	2.47853039
1.03225778	1.58283569	2.40788203
0.97165484	1.48990882	2.26651741
0.88337750	1.35454676	2.06059846
0.77083098	1.18197103	1.79806837
0.63806098	0.97838516	1.48836423
0.48961440	0.75076126	1.14209235
0.33038883	0.50660914	0.77067699
0.16547538	0.25373540	0.38599388
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000000436	1.726	2.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.011166637165	0.988956679587	1

PHI(I,R,2)

1.05446001	1.58623925	2.37442751
1.02361152	1.53911875	2.30139559
0.95773204	1.43970910	2.15099280
0.86075624	1.29375264	1.93158428
0.73843763	1.10980730	1.65585155
0.59765949	0.89817790	1.33912714
0.44576226	0.67009128	0.99848124
0.29045150	0.43690470	0.65094977
0.13954367	0.20999166	0.31287855
0.0	0.0	0.0

PHI(I,2,Z)

1.05446001	1.58623925	2.37442751
1.02681679	1.54396562	2.30880325
0.96828631	1.45563829	2.17511505
0.88167255	1.32525998	1.97909324
0.77048076	1.15800721	1.72832736
0.63882762	0.96002159	1.43189729
0.49116511	0.73811097	1.10010893
0.33206853	0.49925588	0.74381123
0.16666796	0.25071539	0.37348334
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0044849251	1.726	2.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
20	1.011080701415	0.989040734929	1

PHI(I,R,2)

1.05231831	1.58350836	2.35972128
1.02142745	1.53719778	2.29066128
0.95599455	1.43889650	2.14423809
0.85970785	1.29405181	1.92851346
0.73806324	1.11094662	1.65571420
0.59769491	0.89964370	1.34084408
0.44603211	0.67134982	1.00061902
0.29084567	0.43776395	0.65248619
0.13979396	0.21040947	0.31362800
0.0	0.0	0.0

PHI(I,2,Z)

1.05231831	1.58350836	2.35972128
1.02465535	1.54264527	2.29787580
0.96666417	1.45492243	2.16803232
0.88083409	1.32587166	1.97579285
0.77043111	1.15975709	1.72835037
0.63931664	0.96240843	1.43434546
0.49135294	0.74042335	1.10358876
0.33278830	0.50096994	0.74675736
0.16712027	0.25158016	0.37508055
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0004472154	1.726	2.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.010998298877	0.989121347792	1

PHI(I,R,2)

1.05113531	1.58199728	2.35804366
1.02064539	1.53609615	2.28962506
0.95559002	1.43818101	2.14367405
0.85956971	1.29366601	1.92826270
0.73806286	1.11079720	1.65566362
0.59774279	0.89962564	1.34090550
0.44608643	0.67138157	1.00070414
0.29088628	0.43779883	0.65254564
0.13981593	0.21043026	0.31364935
0.0	0.0	0.0

PHI(I,2,Z)

1.05113531	1.58199728	2.35804366
1.02389760	1.54098921	2.29691998
0.96635597	1.45437975	2.16782080
0.88090010	1.32576233	1.97610360
0.77071465	1.15995296	1.72891167
0.63965724	0.96272772	1.43495604
0.49215204	0.74072275	1.10405763
0.33299156	0.50117353	0.74700677
0.16722639	0.25168556	0.37514130
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000452138	1.726	2.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
40	1.010994859840	0.989124712422	1

PHI(I,R,2)

1.05109642	1.58196419	2.35793856
1.02061552	1.53608458	2.28955555
0.95556596	1.43817898	2.14362633
0.85955115	1.29367038	1.92823435
0.73804741	1.11080036	1.65566400
0.59773817	0.89962740	1.34090770
0.44608648	0.67138356	1.00070701
0.29088750	0.43780092	0.65254873
0.13981684	0.21043163	0.31365151
0.0	0.0	0.0

PHI(I,2,Z)

1.05109642	1.58196419	2.35793856
1.02387039	1.54098419	2.29685846
0.96634091	1.45439609	2.16779806
0.88089834	1.32579948	1.97612313
0.77972553	1.15998322	1.72897166
0.63968180	0.96275537	1.43500072
0.49217250	0.74074603	1.10409258
0.33300437	0.50118952	0.74703015
0.16723199	0.25169321	0.37515253
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000035159	1.726	2.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
50	1.010995164073	0.989124414771	1

PHI(I,R,2)

1.05109723	1.98195634	2.35792887
1.02061524	1.53607940	2.28954863
0.95556517	1.43817564	2.14362165
0.85955032	1.29366827	1.92823132
0.73804676	1.11079905	1.65566208
0.59773773	0.89962663	1.34090651
0.44608622	0.67138316	1.00070632
0.29088738	0.43780076	0.65254837
0.13981679	0.21043160	0.31365135
0.0	0.0	0.0

PHI(I,2,Z)

1.05109723	1.98195634	2.35792887
1.02287098	1.54097946	2.29685223
0.96634115	1.45439408	2.16779546
0.88089912	1.32579936	1.97612319
0.77072673	1.15998410	1.72897316
0.63968308	0.96275656	1.43500260
0.49217357	0.74074703	1.10409418
0.33300511	0.50119016	0.74703119
0.16723238	0.25169351	0.37515282
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000000989	1.726	2.5%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
60	1.010995148385	0.989124430120	1

PHI(I,R,2)

1.05109698	1.58195605	2.35792840
1.02061508	1.53607920	2.28954828
0.95556506	1.43817550	2.14362142
0.85955025	1.29366819	1.92823117
0.73804672	1.11079900	1.65566200
0.59773772	0.89962660	1.34090648
0.44608622	0.67138315	1.00070630
0.29088738	0.43780076	0.65254837
0.13981679	0.21043160	0.31365135
0.0	0.0	0.0

PHI(I,2,Z)

1.05109698	1.58195605	2.35792840
1.02387084	1.54097928	2.29685194
0.96634111	1.45439403	2.16779537
0.88089917	1.32579942	1.97612324
0.77072680	1.15998423	1.72897330
0.63968316	0.96275669	1.43500279
0.49217365	0.74074714	1.10409434
0.33380516	0.50119022	0.74703129
0.16723241	0.25169354	0.37515286
0.0	0.0	0.0

SAVE	W	VOL. U235
0.000000C103	1.726	2.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.0C8532314174	0.99153e870310	2

PHI(I,R,2)

1.04323420	1.54067748	2.26063293
1.01549300	1.49950053	2.19724334
0.95265701	1.40588804	2.05919022
0.85845765	1.26668361	1.85410462
0.73840614	1.08943477	1.59367890
0.59920158	0.88398772	1.29227874
0.44808300	0.66121175	0.96610355
0.29273247	0.43223160	0.63149779
0.14100917	0.20828499	0.30432291
0.0	0.0	0.0

PHI(I,2,Z)

1.04323420	1.54067748	2.26063293
1.01853826	1.50352445	2.20401755
0.96300034	1.42121789	2.08193254
0.87916584	1.29731324	1.89935596
0.77031129	1.13655758	1.66311031
0.64636287	0.94470019	1.38153085
0.49363225	0.72821533	1.06422955
0.33461136	0.49383519	0.72145096
0.16838499	0.24863427	0.36320523
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0041390406	1.726	3.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
20	1.008449249200	0.991621542476	2

PHI(I,R,2)

1.04114945	1.53780503	2.24720513
1.01336613	1.49692014	2.18743008
0.95096136	1.40489078	2.05301341
0.85743074	1.26678514	1.85130699
0.73803607	1.09038592	1.59358317
0.59923440	0.88536234	1.29389658
0.44834773	0.66237314	0.96810170
0.29311789	0.43303769	0.63292725
0.14125340	0.20868083	0.30501899
0.0	0.0	0.0

PHI(I,2,Z)

1.04114945	1.53780503	2.24720513
1.01643344	1.50144179	2.19402887
0.96141529	1.42030373	2.07545768
0.87834000	1.29769418	1.89635453
0.77025339	1.13806506	1.66317415
0.64083361	0.94686578	1.38384047
0.49430255	0.73036165	1.06749287
0.33531560	0.49544883	0.72420622
0.16882840	0.24945709	0.36469686
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0004139244	1.726	3.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.008371621975	0.991697880233	2

PHI(I,R,2)

1.04000617	1.53635724	2.24559678
1.01260261	1.49586551	2.18640979
0.95055971	1.40420930	2.05243735
0.85728737	1.26642167	1.85103744
0.73802919	1.09024941	1.59351686
0.59927805	0.88529142	1.29394693
0.44840015	0.66240900	0.96818098
0.29315800	0.43307491	0.63298501
0.14127528	0.20870244	0.30504076
0.0	0.0	0.0

PHI(I,2,Z)

1.04000617	1.53635724	2.24559678
1.01569323	1.50042979	2.19308228
0.96110706	1.41978694	2.07520713
0.87839568	1.29759886	1.89660617
0.77052431	1.13826610	1.66368212
0.64116548	0.94718781	1.38441266
0.49459754	0.73066437	1.06794358
0.33551797	0.49565588	0.72445419
0.16893412	0.24956317	0.36476342
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000458630	1.726	3.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
40	1.0008367863140	0.991701576929	2

PHI(I,R,2)

1.03996072	1.53631577	2.24548777
1.01256853	1.49584612	2.18633788
0.95053351	1.40420107	2.05238916
0.85726808	1.26642131	1.85100944
0.73801398	1.0902498C	1.59351604
0.59927351	0.88529185	1.29394822
0.44840018	0.66241039	0.96818330
0.29315920	0.43307674	0.63298780
0.14127618	0.20870372	0.30504278
0.0	0.0	0.0

PHI(I,2,Z)

1.03996072	1.53631577	2.24548777
1.01566253	1.50041691	2.19301856
0.96109081	1.41979737	2.07518471
0.87839468	1.29763159	1.89662649
0.77053705	1.13829592	1.66374048
0.64119141	0.94721671	1.38445789
0.49461922	0.73068919	1.06797988
0.33553170	0.49567304	0.72447881
0.16894020	0.24957140	0.36477529
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000035405	1.726	3.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
50	1.000368107259	0.991701336844	2

PHI(I,R,2)

1.03996068	1.53630791	2.24547733
1.01256764	1.49584090	2.18633049
0.95053233	1.40419770	2.05238421
0.85726702	1.26641918	1.85100626
0.73801320	1.09024850	1.59351407
0.59927302	0.88529110	1.29394704
0.44839991	0.66241002	0.96818265
0.29315908	0.43307660	0.63298748
0.14127613	0.20870371	0.30594265
0.0	0.0	0.0

PHI(I,2,Z)

1.03996068	1.53630791	2.24547733
1.01566206	1.50041221	2.19301194
0.96109083	1.41979552	2.07518209
0.87839551	1.29763179	1.89662682
0.77053845	1.13829723	1.66374243
0.64119294	0.94721834	1.38446029
0.49462052	0.73069058	1.06798195
0.33553260	0.49567395	0.72448017
0.16894066	0.24957183	0.36477575
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000001291	1.726	3.0%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
60	1.0068368089795	0.9917U1354019	2

PHI(I,R,2)

1.03996037	1.53630753	2.24547675
1.01256744	1.49584063	2.18633007
0.95053220	1.40419752	2.05238392
0.85726693	1.26641907	1.85100608
0.73801316	1.09024843	1.59351397
0.59927300	0.88529196	1.29394699
0.44839991	0.66241000	0.96818263
0.29315908	0.43307669	0.63298748
0.14127613	0.20870371	0.30504265
0.0	0.0	0.0

PHI(I,2,Z)

1.03996037	1.53630753	2.24547675
1.01566188	1.50041198	2.19301158
0.96109078	1.41979546	2.07518197
0.87839556	1.29763186	1.89662689
0.77053855	1.13829739	1.66374262
0.64119305	0.94721851	1.38446053
0.49462062	0.73069071	1.06798215
0.33553267	0.49567404	0.72448030
0.16894069	0.24957187	0.36477581
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0002600127	1.726	3.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.005972494460	0.994062964452	3

PHI(I,R,2)

1.03227593	1.49734569	2.15566035
1.00750046	1.46073344	2.10096319
0.94759459	1.37353156	1.97415369
0.85608909	1.24070717	1.78219363
0.73825546	1.06982235	1.53586641
0.60060768	0.87028224	1.24864080
0.45028160	0.65260564	0.93589582
0.29492665	0.42768409	0.61332024
0.14243119	0.20661599	0.29631668
0.0	0.0	0.0

PHI(I,2,Z)

1.03227593	1.49734569	2.15566035
1.01039111	1.46495295	2.10715867
0.95773244	1.38829081	1.99562291
0.87659189	1.27049556	1.82535157
0.77002032	1.11590735	1.60246563
0.64175369	0.92989955	1.33461221
0.49596214	0.71862177	1.03075276
0.33705120	0.48855856	0.70055569
0.17004808	0.24659835	0.35358701
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0037841887	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
20	1.0C5886761717	0.994147689441	3

PHI(I,R,2)

1.03019394	1.49434768	2.14341132
1.00539654	1.45850957	2.09199179
0.94592265	1.37236728	1.96850029
0.85507797	1.24062961	1.77963873
0.73789265	1.07960270	1.53580055
0.60064603	0.87145894	1.25015776
0.45054868	0.65367819	0.93775739
0.29530666	0.42844202	0.61464857
0.14267044	0.20699202	0.29696328
0.0	0.0	0.0

PHI(I,2,Z)

1.03019394	1.49434768	2.14341132
1.00830757	1.46272431	2.09802900
0.95616413	1.38719785	1.98970204
0.87577093	1.27066846	1.82262134
0.76995638	1.11719626	1.60256428
0.64221538	0.93186523	1.33679039
0.49662677	0.72061758	1.03381090
0.33774805	0.49008144	0.70313187
0.17048614	0.24738328	0.35498022
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0003868757	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.005813659462	0.994219943616	3

PHI(I,R,2)

1.02908912	1.49295887	2.14186100
1.00465138	1.45749869	2.09098431
0.94552378	1.37171730	1.96791426
0.85492922	1.24028663	1.77935439
0.73787925	1.07047785	1.53572218
0.60068556	0.87145427	1.25019953
0.45059929	0.65371759	0.93783221
0.29534628	0.42848124	0.61470524
0.14269217	0.20701435	0.29698570
0.0	0.0	0.0

PHI(I,2,Z)

1.02908912	1.49295887	2.14186100
1.00758497	1.46175351	2.09708929
0.95585675	1.38670523	1.98941689
0.87581708	1.27058592	1.82282153
0.77021516	1.11740173	1.60302646
0.64253864	0.93218926	1.33732969
0.49691734	0.72092289	1.03424650
0.33794893	0.49029142	0.70337958
0.17059091	0.24749010	0.35505314
0.0	0.0	0.0

SAVE	W	VOL. U235
0.000459159	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
40	1.0[illegible]9651052	0.994223906038	3

PHI(I,R,2)

1.02903771	1.49290944	2.14174898
1.00461342	1.45747188	2.09091021
0.94549558	1.37170323	1.96786532
0.85490921	1.24028187	1.77932631
0.73786429	1.07047565	1.53571981
0.60068106	0.87145348	1.25019975
0.45059926	0.65371843	0.93783392
0.29534742	0.42848283	0.61470770
0.14269305	0.20701556	0.29698757
0.0	0.0	0.0

PHI(I,2,Z)

1.02903771	1.49290944	2.14174898
1.00755060	1.46173318	2.09702350
0.95583940	1.38671006	1.98939426
0.87581683	1.27061463	1.82284203
0.77022966	1.11743114	1.60308277
0.64256574	0.93221935	1.33737525
0.49694011	0.72094916	1.03428399
0.33796354	0.49030969	0.70340531
0.17359746	0.24749888	0.35506558
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000035227	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
50	1.005809829851	0.994223729299	3

PHI(I,R,2)

1.02903675	1.49290151	2.14173788
1.00461190	1.45746657	2.09090241
0.94549400	1.37169977	1.96786012
0.85490792	1.24027968	1.77932299
0.73784339	1.07047432	1.53571778
0.60068051	0.87145274	1.25019857
0.45359897	0.65271807	0.93783330
0.29534730	0.42648270	0.61470742
0.14269301	0.20701555	0.29698747
0.0	0.0	0.0

PHI(I,2,Z)

1.02903675	1.49290151	2.14173788
1.00754954	1.46172844	2.09701654
0.95583918	1.38670833	1.98939162
0.87581770	1.27061510	1.82284262
0.77023126	1.11743285	1.60308517
0.64256753	0.93222141	1.33737816
0.49694165	0.72095093	1.03428651
0.33796460	0.49031087	0.70340698
0.17059799	0.24749943	0.35506623
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000001621	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
60	1.005809810469	0.994223748458	3

PHI(I,R,2)

1.02903638	1.49290105	2.14173720
1.00461165	1.45746624	2.09090191
0.94549383	1.37169954	1.96785977
0.85490781	1.24027954	1.77932276
0.73786333	1.07047424	1.53571765
0.60068049	0.87145270	1.25019850
0.45059896	0.65371805	0.93783327
0.29534730	0.42848270	0.61470741
0.14269301	0.20701556	0.29698748
0.0	0.0	0.0

PHI(I,2,Z)

1.02903638	1.49290105	2.14173720
1.00754933	1.46172815	2.09701611
0.95583912	1.38670824	1.98939148
0.87581776	1.27061518	1.82284271
0.77023138	1.11743303	1.60308541
0.64256767	0.93222161	1.33737846
0.49694178	0.72095110	1.03428675
0.33796469	0.49031098	0.70340715
0.17059803	0.24749948	0.35506630
0.0	0.0	0.0

SAVE	W	VOL. U235
0.000000C0156	1.726	3.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.003476717883	0.996535327804	4

PHI(I,R,2)

1.02152265	1.45605867	2.05860055
0.99960080	1.42416764	2.01174180
0.94253304	1.34252527	1.89518576
0.85365410	1.21574188	1.71527939
0.73799908	1.05091652	1.48196686
0.60189641	0.85702927	1.20788181
0.45237416	0.64425642	0.90763305
0.29704273	0.42325618	0.59628740
0.14381315	0.20498362	0.28880500
0.0	0.0	0.0

PHI(I,2,Z)

1.02152265	1.45605867	2.05860055
1.00234599	1.42809986	2.01740860
0.95246888	1.35674104	1.91548085
0.87395229	1.24472402	1.75649666
0.76962027	1.09600206	1.54593138
0.64301908	0.91558729	1.29079304
0.49817521	0.70931363	0.99943362
0.33940182	0.48342004	0.68097685
0.17166324	0.24460693	0.34456281
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0034695692	1.726	4.0%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
20	1.003389211798	0.996622236159	4

PHI(I,R,2)

1.01944755	1.45296926	2.04738852
0.99752087	1.42183391	2.00350951
0.94088437	1.34122513	1.88998929
0.85265772	1.21551330	1.71293245
0.73764231	1.05154925	1.48192019
0.60193881	0.85808480	1.20930253
0.45264237	0.64525002	0.90936845
0.29741700	0.42397080	0.59752326
0.14404745	0.20534160	0.28940665
0.0	0.0	0.0

PHI(I,2,Z)

1.01944755	1.45296926	2.04738852
1.00027906	1.42575839	2.00903317
0.95091755	1.35550196	1.91004381
0.87313613	1.24472032	1.75399896
0.76955008	1.09710076	1.54605227
0.64347132	0.91737609	1.29284581
0.49383323	0.71117467	1.00230199
0.34009074	0.48486078	0.68338896
0.17209575	0.24535717	0.34586638
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0003630457	1.726	4.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.003320158376	0.996690828597	4

PHI(I,R,2)

1.01837767	1.45163613	2.04589303
0.99679228	1.42086262	2.00251701
0.94048785	1.34066344	1.88939768
0.85259391	1.21518801	1.71263738
0.73762290	1.05143395	1.48183253
0.60197458	0.85809499	1.20933736
0.45269136	0.64529207	0.90943956
0.29745622	0.42401153	0.59757903
0.14406907	0.20536449	0.28942965
0.0	0.0	0.0

PHI(I,2,Z)

1.01837767	1.45163613	2.04589303
0.99957321	1.42482576	2.00810314
0.95061058	1.35503075	1.90973099
0.87317315	1.24464789	1.75415549
0.76979741	1.09730844	1.54647457
0.64378648	0.91770028	1.29335584
0.49911967	0.71148109	1.00272376
0.34029031	0.48507272	0.68363603
0.17219974	0.24546444	0.34594469
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000456737	1.726	4.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
40	1.003315956915	0.996695002315	4

PHI(I,R,2)

1.01832094	1.45157949	2.04577864
0.99675078	1.42082914	2.00244090
0.94345775	1.34658415	1.88934781
0.85248318	1.21517939	1.71260894
0.73760809	1.05142954	1.48182859
0.60197307	0.85808317	1.20933655
0.45269126	0.64529245	0.90944070
0.29745730	0.42401293	0.59758119
0.14406993	0.20536563	0.28943138
0.0	0.0	0.0

PHI(I,2,Z)

1.01832094	1.45157949	2.04577864
0.99953549	1.42479872	2.00803541
0.95059215	1.35503059	1.90970779
0.87317351	1.24467308	1.75417570
0.76981346	1.09733755	1.54652883
0.64381466	0.91773143	1.29340162
0.49914350	0.71150872	1.00276229
0.34030578	0.48509206	0.68366278
0.17220675	0.24547374	0.34595768
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000034973	1.726	4.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
50	1.003316073904	0.996694886098	4

PHI(I,R,2)

1.01831909	1.45157141	2.04576693
0.99674863	1.42082366	2.00243270
0.94045378	1.34058054	1.88934235
0.85248165	1.21517710	1.71260546
0.73760707	1.05142816	1.48182648
0.60196946	0.85808241	1.20933535
0.45269095	0.64529210	0.90944009
0.29745717	0.42401282	0.59758093
0.14406989	0.20536563	0.28943131
0.0	0.0	0.0

PHI(I,2,Z)

1.01831909	1.45157141	2.04576693
0.99953385	1.42479387	2.00802810
0.95059170	1.35502890	1.90970509
0.87317440	1.24467377	1.75417650
0.76981526	1.09733961	1.54653163
0.64381672	0.91773394	1.29340501
0.49914529	0.71151085	1.00276525
0.34030701	0.48509350	0.68366476
0.17220735	0.24547442	0.34595849
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000091946	1.726	4.0%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
60	1.003316052437	0.996694907423	4

PHI(I,R,2)

1.01831866	1.45157085	2.04576613
0.99674833	1.42082325	2.00243212
0.94045557	1.34058026	1.88934194
0.85248152	1.21517692	1.71260520
0.73760699	1.05142805	1.48182633
0.60196942	0.85808235	1.20933526
0.45269093	0.64529208	0.90944006
0.29745717	0.42401281	0.59758092
0.14406989	0.20536563	0.28943131
0.0	0.0	0.0

PHI(I,2,Z)

1.01831866	1.45157085	2.04576613
0.99953359	1.42479352	2.00802760
0.95059162	1.35502879	1.90970493
0.87317447	1.24467387	1.75417662
0.76981541	1.09733982	1.54653192
0.64381689	0.91773418	1.29340536
0.49914545	0.71151106	1.00276554
0.34030712	0.48509363	0.68366496
0.17220741	0.24547449	0.34595858
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000000190	1.726	4.0%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
10	1.0010041235645	0.998959847399	5

PHI(I,R,2)

1.01096846	1.41667323	1.96861887
0.99179420	1.38918867	1.92884617
0.93747704	1.31278219	1.82166559
0.85116011	1.19172568	1.65285801
0.73764551	1.03267626	1.43159108
0.60307582	0.84420422	1.16971920
0.45436727	0.63615122	0.88112668
0.29908524	0.41894258	0.58028950
0.14515726	0.20338660	0.28174094
0.0	0.0	0.0

PHI(I,2,Z)

1.01096846	1.41667323	1.96861887
0.99438790	1.39284945	1.93402874
0.94721427	1.32647977	1.84087498
0.87125463	1.21993444	1.69227160
0.76912007	1.07679855	1.49309850
0.64416804	0.90173705	1.24976894
0.50027879	0.70027680	0.97066285
0.34166845	0.47841364	0.66258820
0.17323312	0.24265852	0.33607624
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0031895268	1.726	4.5%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
20	1.000952694475	0.999048212288	5

PHI(I,R,2)

1.00890364	1.41352054	1.95832309
0.98973902	1.38677637	1.92126576
0.93585119	1.31137224	1.81687001
0.85317755	1.19136975	1.65069023
0.73729373	1.03318110	1.43155544
0.60312103	0.84515292	1.17104858
0.45463561	0.63707436	0.88274554
0.29945355	0.41961807	0.58144077
0.14538668	0.20372809	0.28230168
0.0	0.0	0.0

PHI(I,2,Z)

1.00890364	1.41352054	1.95832309
0.99235031	1.39042263	1.92631860
0.94568007	1.32512193	1.83586259
0.87044320	1.21978075	1.68997432
0.76904348	1.07773129	1.49323194
0.64461029	0.90336877	1.25170256
0.50092943	0.70201667	0.97275561
0.34234901	0.47977974	0.66484978
0.17365996	0.24337685	0.33729798
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0003419076	1.726	4.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
30	1.000887280566	0.999113506003	5

PHI(I,R,2)

1.00786564	1.41223981	1.95687962
0.98902546	1.38583743	1.92028994
0.93545666	1.31077597	1.81627633
0.85001900	1.19105975	1.65038769
0.73726876	1.03307362	1.43146068
0.60315335	0.84515681	1.17107774
0.45468314	0.63711834	0.88281349
0.29949243	0.41965992	0.58149573
0.14540822	0.20375138	0.28232516
0.0	0.0	0.0

PHI(I,2,Z)

1.00786564	1.41223981	1.95687962
0.99165790	1.38952534	1.92540049
0.94537318	1.32466970	1.83552782
0.87047149	1.21971615	1.69009368
0.76928001	1.07793935	1.49361931
0.64491781	0.90369162	1.25218636
0.50121200	0.70232301	0.97316459
0.34254740	0.47999284	0.66509584
0.17376328	0.24348433	0.33738085
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000452160	1.726	4.5%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
40	1.000882932399	0.999117846483	5

PHI(I,R,2)

1.00780415	1.41217667	1.95676334
0.98898071	1.38579796	1.92021198
0.93542475	1.31075203	1.81622542
0.84999752	1.19104773	1.65035869
0.73725402	1.03306728	1.43145520
0.60314878	0.84515411	1.17107594
0.45468294	0.63711835	0.88281411
0.29949344	0.41966116	0.58149762
0.14540906	0.20375247	0.28232678
0.0	0.0	0.0

PHI(I,2,Z)

1.00780415	1.41217667	1.95676334
0.99161710	1.38949226	1.92533092
0.94535367	1.32466508	1.83550379
0.87047231	1.21973826	1.69011326
0.76929741	1.07796824	1.49367155
0.64494698	0.90372387	1.25223225
0.50123686	0.70235192	0.97320404
0.34256371	0.48001318	0.66512351
0.17377075	0.24349414	0.33739434
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000036599	1.726	4.5%

OUTPUT DATA

ITER	SCALE	SKEFF	OUTER ITER.
50	1.000882991283	0.999117787703	5

PHI(I,R,2)

1.00780142	1.41216837	1.95675107
0.98897796	1.38579226	1.92320340
0.93542238	1.31074823	1.81621970
0.84999576	1.19104531	1.65035505
0.73725287	1.03306583	1.43145301
0.60314811	0.84515332	1.17107471
0.45468260	0.63711799	0.88281350
0.29949332	0.41966105	0.58149738
0.14540902	0.20375247	0.28232672
0.0	0.0	0.0

PHI(I,2,Z)

1.00780142	1.41216837	1.95675107
0.99161489	1.38948724	1.92532329
0.94535298	1.32466337	1.83550101
0.87047324	1.21973913	1.69011423
0.76929940	1.07797062	1.49367470
0.64494931	0.90372671	1.25223608
0.50123889	0.70235446	0.97320741
0.34256512	0.48001487	0.66512579
0.17377144	0.24349495	0.33739532
0.0	0.0	0.0

SAVE	W	VOL. U235
0.0000002265	1.726	4.5%

OUTPUT DATA

ITER	SCALF	SKEFF	OUTER ITER.
60	1.000882967579	0.999117811365	5

PHI(I,R,2)

1.00780092	1.41216772	1.95675016
0.98897760	1.38579178	1.92020272
0.93542213	1.31074790	1.81621923
0.84999560	1.19104509	1.65035474
0.73725277	1.03306569	1.43145282
0.60314806	0.84515325	1.17107461
0.45468258	0.63711795	0.88281345
0.29949331	0.41966104	0.58149737
0.14540902	0.20375247	0.28232672
0.0	0.0	0.0

PHI(I,2,Z)

1.00780092	1.41216772	1.95675016
0.99161458	1.38948683	1.92532271
0.94535289	1.32466324	1.83550082
0.87047332	1.21973923	1.69011436
0.76929958	1.07797087	1.49367504
0.64494952	0.90372699	1.25223648
0.50123909	0.70235465	0.97320775
0.34256526	0.48001584	0.66512603
0.17377151	0.24349502	0.33739543
0.0	0.0	0.0

SAVF	W	VOL. U235
0.00000030226	1.726	4.5%

```
C      SOLUTION OF NEUTRON DIFFUSION EQUATIONS IN SPACE AND ENERGY USING
C     6  FEWGROUP ( 3 GROUP) FINITE DIFFERENCE TECHNIQUE
       IMPLICIT REAL*8(A-H,O-Z)
       INTEGER Q,C,R,Z,  RMIN1,ZMIN1,F,RMIN2,EPR
       REAL LABEL1/4H    /,LABEL2/4H    /,LABEL3/4H    /,LABEL4/4H    /,
      6YPHIR,YPHIZ,XX,YY,VBES,YYY
       REAL*8 NU
       DIMENSION A( 3, 5),  CHI( 3),SIGTR(7, 3),SIGF(3, 3),SIGC(7, 3),SIG
      6IN(7, 3, 5),NU(3, 3),  SUMR(4,20,20),SUML(3,20,20),        H(7),SSIG
      6TR( 5),SSIGF( 5),SSIGC( 5),SSIGIN( 3, 5),             HH(7),SNU( 4),
      6BETA(40),ALPHA(40),DELE( 3),T(15)
       DIMENSION W235(20)
       DIMENSION W238(20)
       DIMENSION HH235(20)
       DIMENSION HH238(20)
       DIMENSION S(3)
C      ALL DIMENSION STATEMENTS BELOW ARE GRID SPACE DEPENDENT
       DIMENSION PHI( 3,21,21)
       DIMENSION PHO(27)
       DIMENSION FLXRZ(21,21)
       DIMENSION FLXR(27)
       DIMENSION FLXI(350)
       DIMENSION VBES(27)
C      "GS" IS GRID SPACING IN THE 'R' AND 'Z' DIRECTIONS IN CM.
       GS=2.0
C      "GSSQ" IS GRID SPACING IN CENTIMETERS SQUARED
       GSSQ=4.0
C      "C" INDICATES NUMBER OF ELEMENTAL COMPONENTS IN CORE COMPOSITION
       C=5
       R=20
       Z=20
       RMIN1=R-1
       RMIN2=R-2
       ZMIN1=Z-1
C      SET THE FLUX MATRIX TO ZERO
       DO 81 L=1,R
       DO 83 M=1,Z
       DO 84 I=1,3
       PHI(I,L,M  )=0.0
    84 CONTINUE
    83 CONTINUE
    81 CONTINUE
C      ***************************************************************
C      A.INPUT DATA
C      ***************************************************************
C      READ IN SOURCE IF ANY
C      SCALE ORIGINAL FLUX GUESS
       READ(5,10) (S(I),I=1,3)
```

```
C       READ IN ENERGY INTERVALS "DELE(I)"
        READ(5,911)(DELE(I),I=1, 3)
  911   FORMAT(12F6.0)
        WRITE(6,7016)
        WRITE(6,6006)
        WRITE(6,6006)
        WRITE(6,5999)
 5999   FORMAT(' ',50X,'INPUT DATA')
        WRITE(6,6006)
 6006   FORMAT(' ',//////)
        WRITE(6,6000)
 6000   FORMAT(' ',52X,'DELE(I)')
        WRITE(6,6001)
 6001   FORMAT(' ',/)
        WRITE(6, 851)(DELE(I),I=1, 3)
  851   FORMAT(' ',42X,3F8.5)
        WRITE(6,6006)
C       COMPUTE MATRIX OF COEFFICIENTS A(I,J) USING ANL  DATA
C       SOLUTION FOR TYPICAL FAST REACTOR CORE CONTAINING THE FOLLOWING
C     6VOLUME PERCENTAGES:U235, 2%;U238,30%;NA,33%;FE,27%;PU239,8%
        READ(5,10)  (H(I),I=1,C)
        READ(5,10)(((SIGIN(I,J,K),K=1, 5),J=1, 3),I=1,C)
        READ(5,10)((SIGTR(I,J),J=1, 3),I=1,C)
        READ(5,10)(( SIGF(I,J),J=1, 3),I=1,3)
        READ(5,10)(( SIGC(I,J),J=1, 3),I=1,C)
        READ(5,10)(CHI(I),I=1, 3)
        READ(5.10) ((NU(I,J),J=1, 3),I=1,3)
   10   FORMAT (12F6.0)
        WRITE(6,6002)
 6002   FORMAT(' ',54X,'H(I)')
        WRITE(6,6001)
        WRITE(6,850) (H(I),I=1,C)
  850   FORMAT(' ',30X,5F10.6)
        WRITE(6,6006)
        WRITE(6,6003)
 6003   FORMAT(' ',49X,'SIGIN(I,J,K)')
        WRITE(6,6001)
        WRITE(6,852) (((SIGIN(I,J,K),K=1, 5),J=1, 3),I=1,C)
  852   FORMAT(' ',34X,5F8.4)
        WRITE(6,7016)
 7016   FORMAT('1')
        WRITE(6,6006)
        WRITE(6,6006)
        WRITE(6,6004)
 6004   FORMAT(' ',51X,'SIGTR(I,J)')
        WRITE(6,6001)
        WRITE(6,951)((SIGTR(I,J),J=1, 3),I=1,C)
  951   FORMAT(' ',44X,3F8.5)
```

```
      WRITE(6,6006)
      WRITE(6,6005)
 6005 FORMAT(' ',52X,'SIGF(I,J)')
      WRITE(6,6001)
      WRITE(6,951) ((SIGF(I,J),J=1, 3),I=1,3)
      WRITE(6,6006)
      WRITE(6,6007)
 6007 FORMAT(' ',52X,'SIGC(I,J)')
      WRITE(6,6001)
      WRITE(6,951) ((SIGC(I,J),J=1, 3),I=1,C)
      WRITE(6,6006)
      WRITE(6,6008)
 6008 FORMAT(' ',54X,'CHI(I)')
      WRITE(6,6001)
      WRITE(6,951) (CHI(I),I=1, 3)
      WRITE(6,7016)
      WRITE(6,6006)
      WRITE(6,6006)
      WRITE(6,6009)
 6009 FORMAT(' ',53X,'NU(I,J)')
      WRITE(6,6001)
      WRITE(6,951) ((NU(I,J),J=1, 3),I=1,3)
      WRITE(6,6006)
C     ************************************************************
C     B.OUTER ITERATION
C     ************************************************************
      JJJ=0
 2000 GO TO 2001
C     OUTER ITERATION
C     RECOMPUTE N235 AND N238 (H(1) AND H(2))
 2002 JJJ=JJJ+1
      W235(JJJ)=((.005*JJJ)+.02)*19.1
      W238(JJJ)=6.12-W235(JJJ)
      HH235(JJJ)=(W235(JJJ)*.6022)/235.0
      H(1)=HH235(JJJ)
      HH238(JJJ)=(W238(JJJ)*.6022)/238.0
      H(2)=HH238(JJJ)
C     COMPUTE RADIUS"RHO"AT EACH RADIAL GRID POINT
 2001 RHO(2)=0.0
      DO 15 K=2,P
      RHO(K+1)=GS+RHO(K)
   15 CONTINUE
C     ************************************************************
C     C.COMPUTATION OF MATRIX OF COEFFICIENTS
C     ************************************************************
C     COMPUTATION OF MATRIX OF COEFFICIENTS FOR FINITE  DIFF.  SOLUTION
C    60F  3 GROUP DIFFUSION EQUATIONS IN SPACE AND ENERGY
      DO 151 K=1,3
```

```
      HH(1)=0.0
      DO 152 L=1,C
      HH(L+1)=H(L)*SIGTR(L,K)+HH(L)
  152 CONTINUE
      SSIGTR(K)=HH(C+1)
  151 CONTINUE
      DO 153 K=1,3
      HH(1)=0.0
      DO 154 L=1,C
      HH(L+1)=H(L)*SIGC(L,K)+HH(L)
  154 CONTINUE
      SSIGC(K)=HH(C+1)
  153 CONTINUE
      DO 1805 K=1,3
      HH(1)=0.0
      DO 1806 L=1,C
      HH(L+1)=H(L)*SIGIN(L,K,K)+HH(L)
 1806 CONTINUE
      SSIGIN(K,K)=HH(C+1)
 1805 CONTINUE
      DO 159 K=1,3
      HH(1)=0.0
      DO 160 L=1,C
      HH(L+1)=H(L)*SIGIN(L,K,K+1)+HH(L)
  160 CONTINUE
      SSIGIN(K,K+1)=HH(C+1)
  159 CONTINUE
      DO 161 K=1,3
      HH(1)=0.0
      DO 162 L=1,C
      HH(L+1)=H(L)*SIGIN(L,K,K+2)+HH(L)
  162 CONTINUE
      SSIGIN(K,K+2)=HH(C+1)
  161 CONTINUE
      DO 17 I=1,3
   17 A(I,I)=1.0/(3.0*SSIGTR(I))
      DO 175 I=1,3
  175 A(I,I+1)=-(SSIGC(I)+SSIGIN(I,I)+SSIGIN(I,I+1)+SSIGIN(I,I+2)+
     6                                                  SIGF(1,I)*H(1)*(
     61.0                  )+SIGF(2,I)*H(2)*(1.0                )+SIGF(3,I)*
     6H(5)*(1.0                  ))
      N=2
      K=1
   19 DO 18 I=N,3
      J=K
   18 A(I,J)=SSIGIN(J,I)+H(1)*CHI(I)*NU(1,J)*SIGF(1,J)+H(2)*CHI(I)*NU(2,
     6J)*SIGF(2,J)+H(5)*CHI(I)*NU(3,J)*SIGF(3,J)
      K=K+1
```

```
      N=N+1
      IF(N+K-7 )19,20,20
   20 N=1
      K=3
   22 DO 21 J=K,5
      I=N
   21 A(I,J)=H(1)*CHI(I)*NU(1,J-2)*SIGF(1,J-2)+H(2)*CHI(I)*NU(2,J-2)*SIG
     6F(2,J-2)+H(5)*CHI(I)*NU(3,J-2)*SIGF(3,J-2)
      N=N+1
      K=K+1
      IF(N+K-9 )22,23,23
   23 IF(JJJ.GE.1) GO TO 1609
      WRITE(6,6010)
 6010 FORMAT(' ',53X,'A(I,J)')
      WRITE(6,6001)
      WRITE(6,6011) ((A(I,J),J=1,5),I=1,3)
 6011 FORMAT('0',31X,5F10.6)
      WRITE(6,7016)
      WRITE(6,6006)
      WRITE(6,6006)
C     ********** ********************************************************
CC    D.INITIAL FLUX ESTIMATE AND BOUNDARY CONDITIONS
CC    ********************************************************************
C     COMPUTE NORMALIZED BESSEL FUNCTION
 1620 ERR=0
      RESULT=0
      NO=0
      KODE=0
      DO 1409 I=1,RMIN1
      X=(2.405*(I-1))/RMIN2
      CALL BES(NO,X,KODE,RESULT,T,ERR)
      J=I+1
      VBES(J)=RESULT
 1409 CONTINUE
C     SET INITIAL VALUES FOR FLUX IN AT ALL INTERIOR POINTS
      DO235 L=2,RMIN1
      DO236 M=2,ZMIN1
      DO238 I=1,3
      PHI(I,L,M)=S(I)*VBES(L)*DCOS((M-2)*3.141592653589/2.00/(Z-2))
  238 CONTINUE
  236 CONTINUE
  235 CONTINUE
C     SET IN BOUNDARY CONDITIONS
C     SET OUTSIDE FACES EQUAL TO ZERO
      DO211 L=2,R
      DO212 I=1,3
      PHI(I,L,Z  )=0.0
  212 CONTINUE
```

```
  211 CONTINUE
      DO214 M=2,Z
      DO215 I=1,3
      PHI(I,R,M  )=0.0
  215 CONTINUE
  214 CONTINUE
C     SET DERIVATIVE EQUAL TO ZERO ON INTERIOR POINTS
      DO222 L=2,RMIN1
      DO223 I=1,3
      PHI(I,L,1  )=PHI(I,L,3  )
  223 CONTINUE
  222 CONTINUE
      DO226 M=2,ZMIN1
      DO228 I=1,3
      PHI(I,1,M  )=PHI(I,3,M  )
  228 CONTINUE
  226 CONTINUE
      WRITE(6,6012)
 6012 FORMAT(' ',44X,'INITIAL FLUX ESTIMATE')
      WRITE(6,6001)
      WRITE(6,6013)
 6013 FORMAT(' ',49X,'PHI(I,R,2)')
      WRITE(6,6001)
      PRINT 404,((PHI(I,L,2  ),I=1,3 ),L=2,R,2)
      WRITE(6,6014)
 6014 FORMAT(' ',////)
      WRITE(6,6015)
 6015 FORMAT(' ',49X,'PHI(I,2,Z)')
      WRITE(6,6001)
      PRINT 404,((PHI(I,2,M  ),I=1,3 ),M=2,Z,2)
  404 FORMAT(' ',42X,3E8.5)
C     BYPASS "INITIAL"FLUX SETTING ON EACH OUTER ITERATION
C     ****************************************************************
C     E.INITIAL FISSION SOURCE INTEGRATION
C     ****************************************************************
 1609 CONTINUE
C     COMPUTE ENERGY-INTEGRATED FLUX AT ALL SPACIAL POINTS
      DO 1011 L=2,RMIN1
      DO 1012 M=2,ZMIN1
      FLXRZ(L,M)=0.0
      DO 1013 I=1,3
      FLXRZ(L,M)=FLXRZ(L,M)+PHI(I,L,M)**2*(CHI(I)*(NU(1,I)*H(1)*SIGF(1,I
     6)+NU(2,I)*H(2)*SIGF(2,I)+NU(3,I)*H(5)*SIGF(3,I)))*DELE(I)
 1013 CONTINUE
 1012 CONTINUE
 1011 CONTINUE
C     COMPUTE"Z"-INTEGRATED FLUX FOR ALL "R" AND "Z"
      DO 1015 L=2,RMIN1
```

```
      FLXR(L)=0.0
      DO 1016 M=2,ZMIN1
      FLXR(L)=FLXR(L)+FLXRZ(L,M)*GS
 1016 CONTINUE
 1015 CONTINUE
C     COMPUTE"R"-INTEGRATED FLUX FOR ALL "R"
      FLUX0=0.0
      DO 1017 L=2,RMIN1
      FLUX0=FLUX0+FLXR(L)*GS
 1017 CONTINUE
      NN=1
      FLXI(NN)=FLUX0
C     SCALE FACTOR FOR ZEROTH INNER ITERATION
      SCALE=1.000
C     **********************************************************
C     SET IN OVER-RELAXATION FACTOR
C     **********************************************************
C     "W" IS THE OVER RELAXATION FACTOR
      ERR=0
      RESULT=0
      NO=0
      KODE=0
      X=(3.8317*GS)/(((R*GS)-(2.0*GS))+GS)
      CALL BES(NO,X,KODE,RESULT,T,ERR)
      J=1
      VBES(J)=RESULT
      AA=(DCOS((3.14159265358*GS)/((Z*GS)-(2.0*GS)))+VBES(1))**2/4.0
      W=2.0/(1.+DSQRT(1.0-AA))
C     BEGIN ITERATION
      ITER=0
  300 SAVE=0.0
C     **********************************************************
C     F.COMPUTATION OF SOURCE VECTORS
C     **********************************************************
C     RECOMPUTE SOURCE VECTORS FOR EACH GROUP BEFORE EACH ITERATION
      DO 1302 I=1,3
      DO 1303 L=2,RMIN1
      DO 1304 M=2,ZMIN1
      F= 4-I
      IF(F.LT.1) GO TO 113
      SUMR(1,L,M)=0.0
      DO 114 K=1,F
      IPLUS1=I+1
      J=IPLUS1+K
      SUMR(K+1,L,M)=A(I,J)* PHI(J-2,L,M)+SUMR(K,L,M)
  114 CONTINUE
  113 SUMP(1,L,M)=0.0
      IMIN1=I-1
```

```
      IF(IMIN1.LT.1) GO TO 115
      SUML(1,L,M)=0.0
      DO 116 K=1,IMIN1
      J=K
      SUML(K+1,L,M)=A(I,J)* PHI(J,L,M)+SUML(K,L,M)
  116 CONTINUE
  115 SUML(1,L,M)=0.0
 1304 CONTINUE
 1303 CONTINUE
 1302 CONTINUE
C     **************************************************************
C     G.SOLUTION OF DIFFUSION EQUATION
C     **************************************************************
C     SOLVE EACH GROUP EQUATION AT ALL POINTS EXCEPT ZERO BOUNDARIES
      NN=NN+1
      DO 1 I=1,3
      DO 842 M=2,ZMIN1
      PPHI1=PHI(I,3,M)
      PPHI2=PHI(I,1,M)
      PPHI3=PHI(I,2,M+1   )
      PPHI4=PHI(I,2,M-1   )
      CSUMR=SUMR( 5-I,2,M)
      CSUML=SUML(I,2,M)
      A2=-A(I,I+1)
      A1=A(I,I)
      SPHI1=PHI(I,2,M)
      TPHI1=PHI(I,2,M)/SCALF
      PHI(I,2,M)=((A1*((2.0/GSSQ)*(PPHI1+PPHI2)+(1.0/GSSQ)*(PPHI3+PPHI4))
     6+CSUMR+CSUML   )/(A2+A1*(6.0/GSSQ)))*W+(1.0-W)*SPHI1
      PHI(I,2,1)=PHI(I,2,3)
      TPHI2=PHI(I,2,M)
      TEST2=DABS(TPHI2-TPHI1)
      IF(TEST2.LT. SAVE) GO TO 842
      SAVE=TEST2
  842 CONTINUE
      DO 112 L=3,RMIN1
      DO 110 M=2,ZMIN1
      PPHI1=PHI(I,L+1,M   )
      PPHI2=PHI(I,L-1,M   )
      PPHI3=PHI(I,L,M+1   )
      PPHI4=PHI(I,L,M-1   )
      CSUMR=SUMR( 5-I,L,M)
      CSUML=SUML(I,L,M)
      RH=RHO(L)
      A2=-A(I,I+1)
      A1=A(I,I)
      SPHI2=PHI(I,L,M)
      TPHI1=PHI(I,L,M)/SCALF
```

```
      PHI(I,L,M)=((A1*((1./GSSQ)*(PPHI1+PPHI2)+(1.00/(2.0*RH*GS))*(PPHI1
     6-PPHI2)+(1.0/GSSQ)*(PPHI3+PPHI4))+CSUMR+CSUML)/(A2+A1*(4.0/GSSQ)))
     6*W+(1.0-W)*SPHI2
      PHI(I,L,1)=PHI(I,L,3)
      PHI(I,1,M)=PHI(I,3,M)
      TPHI2=PHI(I,L,M)
      TEST3=DABS(TPHI2-TPHI1)
      IF(TEST3.LT. SAVE) GO TO 110
      SAVE=TEST3
  110 CONTINUE
  112 CONTINUE
    1 CONTINUE
C     ******************************************************************
C     H. NTH FISSION SOURCE INTEGRATION (FLUX VECTOR NOT YET SCALED)
C     ******************************************************************
      ITER=ITER+1
C     COMPUTE ENERGY-INTEGRATED FLUX AT ALL SPACIAL POINTS
      DO 1001 L=2,RMIN1
      DO 1002 M=2,ZMIN1
      FLXRZ(L,M)=0.0
      DO 1003 I=1,3
      FLXRZ(L,M)=FLXRZ(L,M)+PHI(I,L,M)**2*(CHI(I)*(NU(1,I)*H(1)*SIGF(1,I
     6)+NU(2,I)*H(2)*SIGF(2,I)+NU(3,I)*H(5)*SIGF(3,I)))*DELE(I)
 1003 CONTINUE
 1002 CONTINUE
 1001 CONTINUE
C     COMPUTE"Z"-INTEGRATED FLUX FOR ALL "R" AND "Z"
      DO 1005 L=2,RMIN1
      FLXR(L)=0.0
      DO 1006 M=2,ZMIN1
      FLXR(L)=FLXR(L)+FLXRZ(L,M)*GS
 1006 CONTINUE
 1005 CONTINUE
C     COMPUTE"R"-INTEGRATED FLUX FOR ALL "R"
      FLUX1=0.0
      DO 1007 L=2,RMIN1
      FLUX1=FLUX1+FLXR(L)*GS
 1007 CONTINUE
      FLXI(NN)=FLUX1
C     ******************************************************************
C     I.COMPUTE SCALE FACTOR AND RENORMALIZE FLUX
C     ******************************************************************
      SCALE=DSQRT(FLXI(NN-1)/FLXI(NN))
C     RENORMALIZE AFTER EACH ITERATION
      DO 843 I=1,3
      DO 844 L=1,RMIN1
      DO 845 M=1,ZMIN1
      PHI(I,L,M)=SCALE    *PHI(I,L,M)
```

```
      IF(PHI(I,L,M).GE..000000000001) GO TO 845
      PHI(I,L,M)=0.0
  845 CONTINUE
  844 CONTINUE
  843 CONTINUE
C     ************************************************************
C     J.NTH FISSION SOURCE INTEGRATION (FLUX VECTOR SCALED)
C     ************************************************************
C     RE-INTEGRATE "RENORMALIZED"FLUX
C     COMPUTE ENERGY-INTEGRATED FLUX AT ALL SPACIAL POINTS
      DO 1018 L=2,RMIN1
      DO 1019 M=2,ZMIN1
      FLXRZ(L,M)=0.0
      DO 1020 I=1,3
      FLXRZ(L,M)=FLXRZ(L,M)+PHI(I,L,M)**2*(CHI(I)*(NU(1,I)*H(1)*SIGF(1,I
     6)+NU(2,I)*H(2)*SIGF(2,I)+NU(3,I)*H(5)*SIGF(3,I)))*DELE(I)
 1020 CONTINUE
 1019 CONTINUE
 1018 CONTINUE
C     COMPUTE"Z"-INTEGRATED FLUX FOR ALL "R" AND "Z"
      DO 1021 L=2,RMIN1
      FLXR(L)=0.0
      DO 1022 M=2,ZMIN1
      FLXR(L)=FLXR(L)+FLXRZ(L,M)*GS
 1022 CONTINUE
 1021 CONTINUE
C     COMPUTE"R"-INTEGRATED FLUX FOR ALL "R"
      FLUX2=0.0
      DO 1023 L=2,RMIN1
      FLUX2=FLUX2+FLXR(L)*GS
 1023 CONTINUE
      FLXI(NN)=FLUX2
C     ************************************************************
C     K.END OF INNER ITERATION
C     ************************************************************
 1801 IF(ITER.GT.60) GO TO 499
      IF(ITER.NE.(ITER/10)*10) GO TO 300
C     ************************************************************
C     L.OUTPUT SECTION
C     ************************************************************
      SKEFF=1.0/SCALF
      WRITE(6,7016)
      WRITE(6,6006)
      WRITE(6,6006)
      WRITE(6,6016)
 6016 FORMAT('1',50X,'OUTPUT DATA')
      WRITE(6,511)
      WRITE(6,6017)
```

```
 6017 FORMAT(' ',27X,'ITER',5X,'SCALE',11X,'SKEFF',12X,'OUTER ITER.')
      WRITE(6,6001)
      WRITE(6,846) ITER,SCALE,SKEFF,JJJ
  846 FORMAT(' ',25X,1I6,2F16.12,9X,1I1)
      WRITE(6,511)
      WRITE(6,6018)
 6018 FORMAT(' ',50X,'PHI(I,R,2)')
      WRITE(6,6001)
      PRINT 510,((PHI(I,L,2),I=1,3 ),L=2,R,2)
      WRITE(6,511)
  511 FORMAT(' ',///)
      WRITE(6,6019)
 6019 FORMAT(' ',50X,'PHI(I,2,Z)')
      WRITE(6,6001)
      PRINT 510,((PHI(I,2,M),I=1,3 ),M=2,Z,2)
  510 FORMAT(' ',37X,3F12.8)
      WRITE(6,511)
  512 FORMAT(' ',////)
      WRITE(6,6020)
 6020 FORMAT(' ',42X,'SAVE',9X,'W',9X,'VOL. U235')
      PU235=2.0+.5*JJJ
      WRITE(6,6001)
      WRITE(6,1421) SAVE,W,PU235
 1421 FORMAT(' ',38X,1F12.10,1F8.3,10X,1F3.1,'%')
      GO TO 300
C     END OF INNER ITERATION
CC    ***************************************************************
CC    M.START NTH OUTER ITERATION AND TEST FOR CRITICALITY
CC    ***************************************************************
C     BEGIN OUTER ITERATION
  499 IF(DABS(1.0-SCALE).LE..0005) GO TO 1490
      IF(JJJ.GE.10)GO TO 1490
      GO TO 2002
C     END OF OUTER ITERATION
 1490 CONTINUE
      STOP
      END
```

LIST OF REFERENCES

1. Meghreblian, R. V., and Holmes, D. K., Reactor Analysis, McGraw-Hill Book Company, 1963.

2. Glasstone, Samuel and Edlund, Milton C., The Elements of Nuclear Reactor Theory, p. 227, D. Van Nostrand Company, Inc., 1952.

3. Ehrlich, R., and Hurwitz, H., Jr., "Multigroup Methods for Neutron Diffusion Problems," Nucleonics, v. 12, p. 23-30, February 1954.

4. Sangren, Ward C., Digital Computers and Nuclear Reactor Calculations, p. 10-13, John Wiley and Sons, Inc., 1960.

5. Argonne National Laboratory ANL-7050, Proceedings of the Conference on the Application of Computing Methods to Reactor Problems, by Margaret Butler and others, p. 125-140, 17-19 May 1965.

6. Malaviya, B., and others, FAST NEUTRON SPECTRA-II, Abstracts of series of papers presented at annual meeting. Transactions American Nuclear Society, v. 13, p. 259-265, July 1970.

7. ANL-5800, Reactor Physics Constants, United States Atomic Energy Commission, 1963.

8. Varga, Richard S., Matrix Iterative Analysis, p. 105-114, Prentice-Hall, Inc., 1962.

9. Smith, Gordon D., Numerical Solution of Partial Differential Equations, p. 44, 140-149, Oxford University Press, 1965.

10. Clark, Melville, Jr., and Hansen, Kent F., Numerical Methods of Reactor Analysis, p. 151-178, Academic Press, 1964.

11. Lamarsh, John R., Introduction to Nuclear Reactor Theory, p. 344-353, Addison-Wesley Publishing Company, Inc., 1966.

12. Campan, J. L., and others, "Physical Research on RAPSODIE," Physics of Fast and Intermediate Reactors III, p. 355-398, Globus, Druck and Verlagsanatalt, Vienna, 1962. (In French)

13. Frankel, Stanley P., "Convergence Rates of Iterative Treatments of Partial Differential Equations," Mathematical Tables and Aids to Computing, v. 4, p. 65-75, 1950.

14. Starling, P. P., "The Numerical Solution of Laplace's Equation," British Journal of Applied Physics, v. 14, p. 603-604, 1963.

15. Atomic Energy Commission Research and Development Report WAPD-TM-364, Numerical Methods and Techniques Used in the Two-Dimensional Neutron-Diffusion Program PDQ-5, by L. A. Hageman, p. 33-53, February 1963.

16. Salvadori, Mario G., and Baron, Melvin L., Numerical Methods in Engineering, 2nd Edition, Prentice-Hall, Inc., 1961.

17. Forsythe, George E. and Wasow, Wolfgang R., Finite-Difference Methods for Partial Differential Equations, p. 242-260, 368-375, John Wiley and Sons, Inc., 1960.

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13. ABSTRACT

This study consists of the development of a computer program to numerically solve the space and energy dependent multigroup neutron diffusion equations in a bare homogeneous fast reactor core or reactor material assembly.

The resulting program is unique in that it was designed for future use by Naval Postgraduate School students undertaking experimental studies in neutron diffusion with limited time to determine numerical solutions for verification of their results.

The equations are solved iteratively in cylindrical geometry using a point successive overrelaxation technique. Convergence between successive iterations was less than 10^{-6} after fifty iterations.

The program was tested using ANL three group data. Flux shapes and energy spectra were determined for a typical fast reactor core composition with and without a centrally located Pu-Be point source and for a solid iron cylinder with a source at its center. The program was also used to determine criticality.

Computation times were from one to ten minutes with less than 150K words of core storage using the IBM 360/67.

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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